

Discrete Probability Distribution

If a variable X can assume a discrete set of values x_1, x_2, \dots, x_K with respective probabilities p_1, p_2, \dots, p_k where

$$p_1 + p_2 + \dots + p_K = 1,$$

we say that a discrete probability distribution for X .

Consider, Two dice are thrown

~~(1,1)~~ ~~(1,2)~~ ~~(1,3)~~ ~~(1,4)~~ ~~(1,5)~~ ~~(1,6)~~
~~(2,1)~~ ~~(2,2)~~ ~~(2,3)~~ ~~(2,4)~~ ~~(2,5)~~ ~~(2,6)~~
~~(3,1)~~ ~~(3,2)~~ ~~(3,3)~~ ~~(3,4)~~ ~~(3,5)~~ ~~(3,6)~~
~~(4,1)~~ ~~(4,2)~~ ~~(4,3)~~ ~~(4,4)~~ ~~(4,5)~~ ~~(4,6)~~
~~(5,1)~~ ~~(5,2)~~ ~~(5,3)~~ ~~(5,4)~~ ~~(5,5)~~ ~~(5,6)~~
~~(6,1)~~ ~~(6,2)~~ ~~(6,3)~~ ~~(6,4)~~ ~~(6,5)~~ ~~(6,6)~~

Let X be the sum of score on the dice

$$P_n = P(x = n)$$

$$P_2 = \frac{1}{36}, \quad P_3 = \frac{2}{36}, \quad P_4 = \frac{3}{36}, \quad P_5 = \frac{4}{36}, \quad P_6 = \frac{5}{36}$$

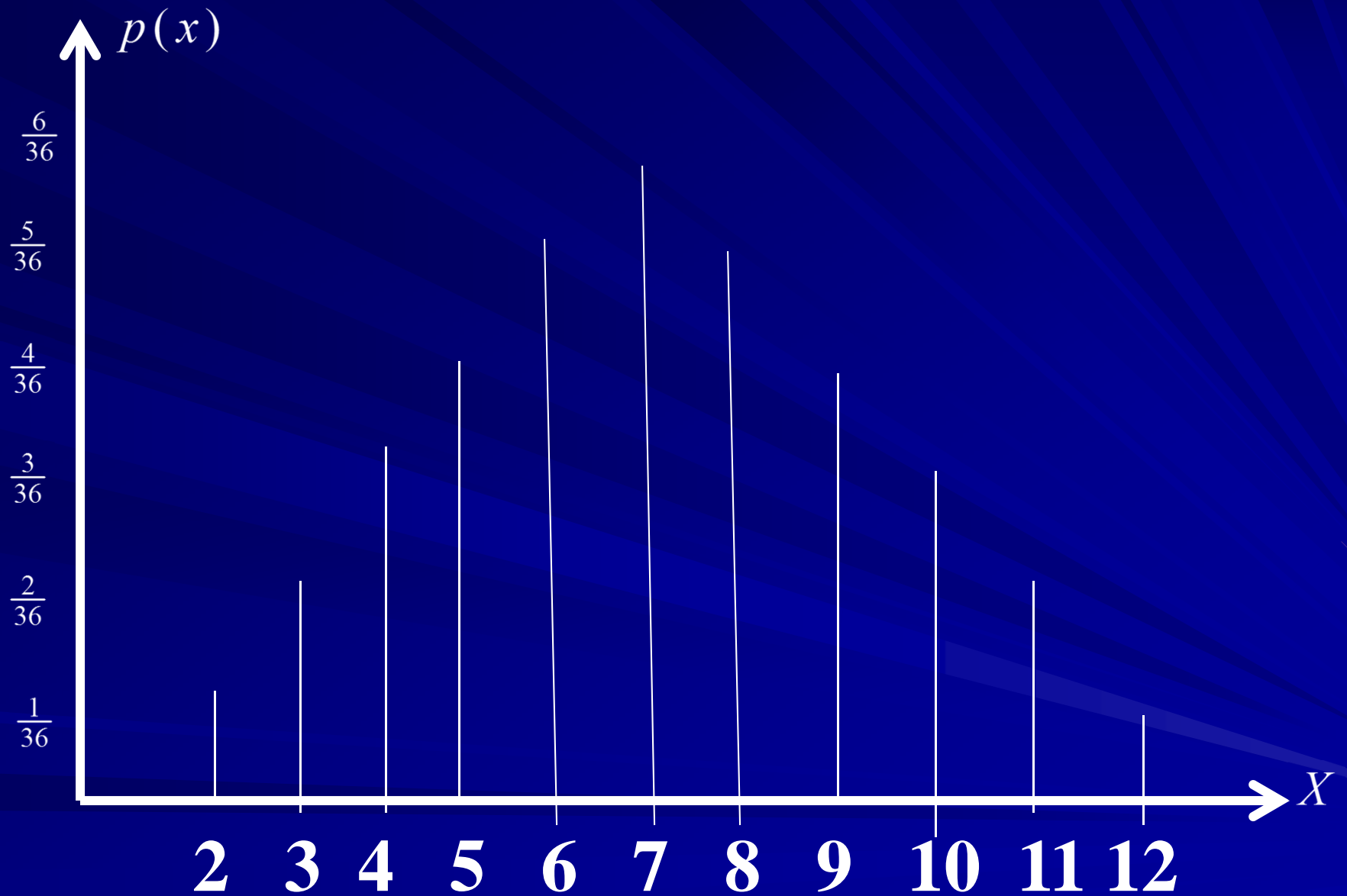
$$P_7 = \frac{6}{36}$$

$$P_8 = \frac{5}{36}, \quad P_9 = \frac{4}{36}, \quad P_{10} = \frac{3}{36}, \quad P_{11} = \frac{2}{36}, \quad P_{12} = \frac{1}{36}$$

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$p(x) = \begin{cases} \frac{j-1}{36} & , j = 2, 3, 4, 5, 6, 7 \\ \frac{13-j}{36} & , j = 8, 9, 10, 11, 12 \\ 0 & , \textit{otherwise} \end{cases}$$

Graph



Mathematical Expectation

(mean , average)

$$\mu = E(X) = p_1 \cdot x_1 + \dots + p_k \cdot x_k$$

$$E(X) = \sum_{j=1}^k p_j \cdot x_j$$

$$E(3X + 5) = \sum_{j=1}^k p_j \cdot (3x_j + 5)$$

$$\mu = E(X^2) = p_1 \cdot x_1^2 + \dots + p_k \cdot x_k^2$$

$$E(X^2) = \sum_{j=1}^k p_j \cdot x_j^2$$

Expected number of the sum of score

$$E(X) = \frac{1}{36} \times 2 + \frac{2}{36} \times 3 + \dots + \frac{1}{36} \times 12$$

$$E(X^2) = \frac{1}{36} \times 2^2 + \frac{2}{36} \times 3^2 + \dots + \frac{1}{36} \times 12^2$$

$$E(X^2) = \frac{1}{36} \times 2^2 + \frac{2}{36} \times 3^2 + \dots + \frac{1}{36} \times 12^2$$

Some Results of $E(X)$

a and b are constant.

$$* E(a) = a$$

$$* E(aX) = aE(X)$$

$$* E(aX + b) = aE(X) + b$$

$$* E[f(x) \pm g(x)] = E[f(x)] \pm E[g(x)]$$

$$e.g, E(3X + 5) = 3E(X) + 5$$

Expected number of getting 7 in 300 times .

Expected number of getting 7 in 300 times

$$= \frac{6}{36} \times 300 = 50$$

Variance

$$Var(X) = E(X - \bar{X})^2$$

$$Var(X) = E(x^2) - \{E(x)\}^2$$

$$\text{Note } E(x^2) \geq \{E(x)\}^2$$

Standard deviation

$$\sigma = \sqrt{Var(x)}$$

$$Var(x) = \sigma^2$$

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

Variance

$$Var(X) = E(x^2) - \{E(x)\}^2$$

$$Note_ E(x^2) \geq \{E(x)\}^2$$

Standard deviation

$$\sigma = \sqrt{Var(x)}$$

$$Var(x) = \sigma^2$$

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

Some Results of $\text{Var}(X)$

a and b are constant.

$$* \text{Var}(a) = 0$$

$$* \text{Var}(aX) = a^2 \text{Var}(X)$$

$$* \text{Var}(aX + b) = a^2 \text{Var}(X)$$

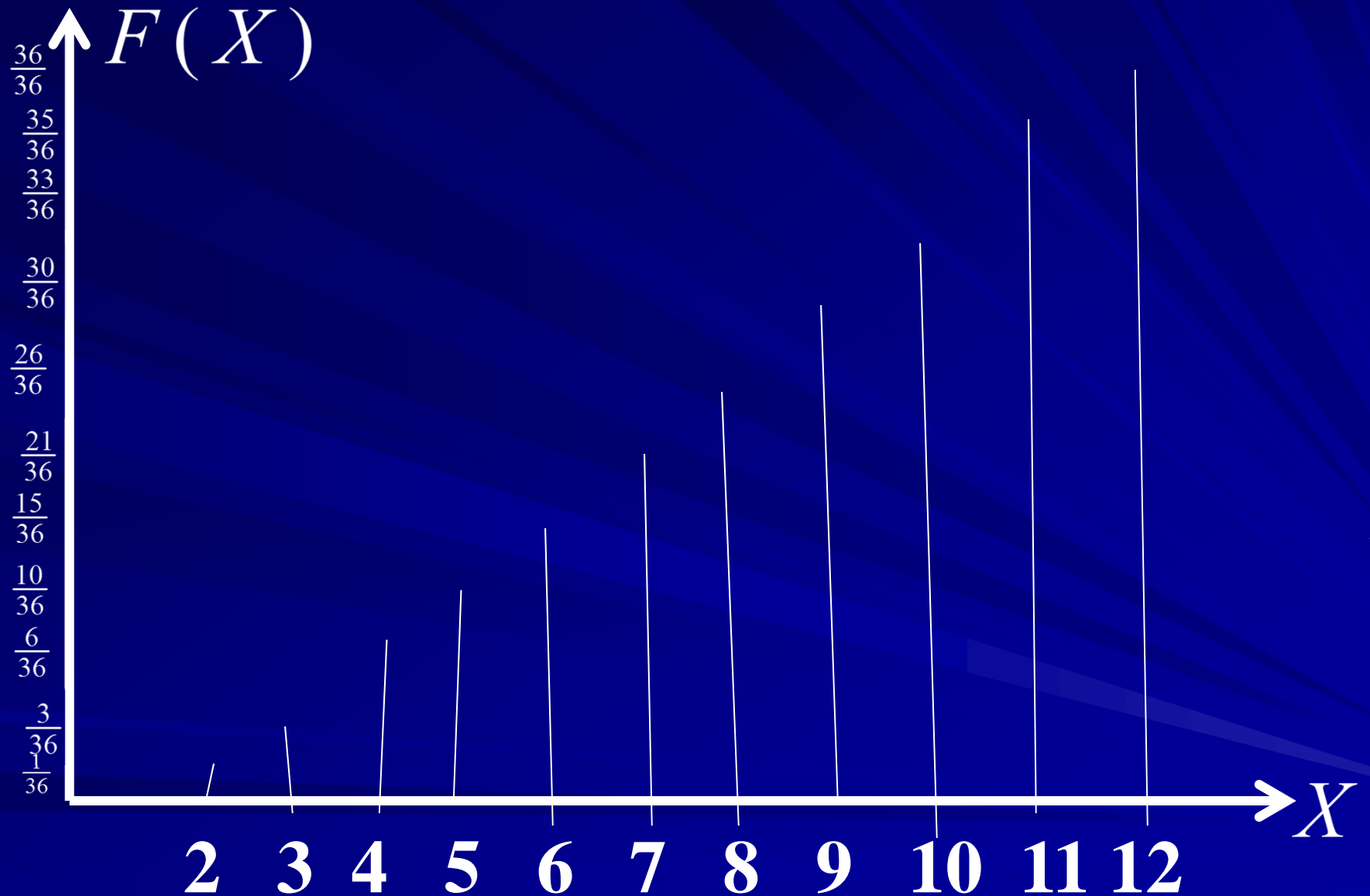
$$* \text{Var}[f(x) \pm g(x)] = \text{Var}[f(x)] \pm \text{Var}[g(x)]$$

$$\text{e.g. , } \text{Var}(3X + 5) = 9 \text{Var}(X)$$

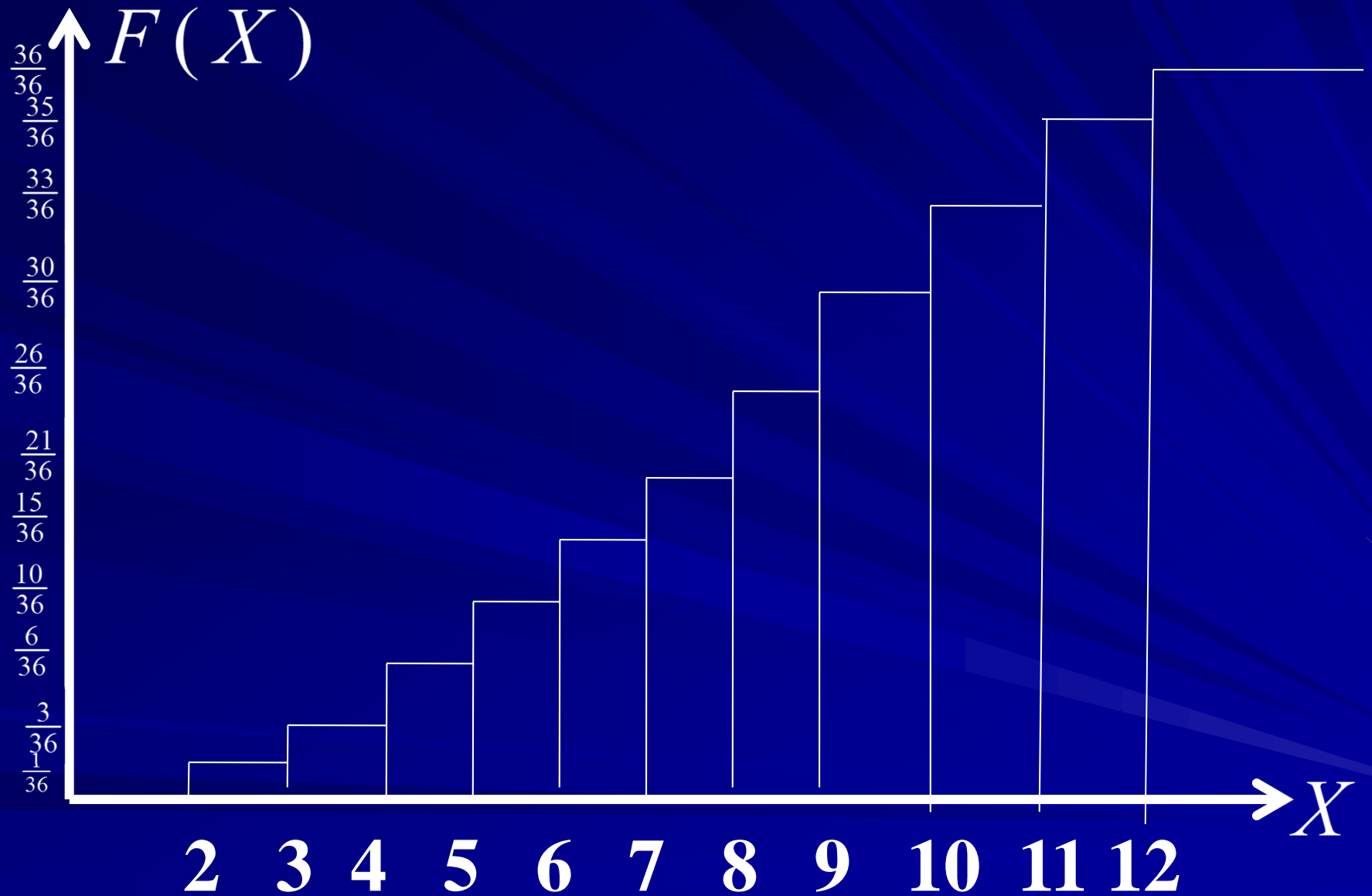
Cumulative Probability, $F(X)$

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
F(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	$\frac{36}{36}$

Cumulative Graph



Cumulative Graph



Median

The median splits the area under the curve $y = f(x)$ into two halves. So if the value of the median is m ,

$$P(a \leq x \leq m) = F(x) = \frac{1}{2} = 0.5$$

$$\text{i.e, } F(m) = \frac{1}{2}$$

Pg 107 . EX (1) No. 1

If X is the random variable showing the number of boy in families with three children construct a table showing the probability distribution of X .

Pg 107 . EX (1) No. 1

X be the numbers of boys in family with three children

$$R_x = \{0, 1, 2, 3\} \quad P(b) = \frac{1}{2}$$

$$P(g, g, g) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(g, g, b) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(g, b, g) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(g, b, b) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(b, g, g) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(b, g, b) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(b, b, g) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(b, b, b) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(x=0) = \frac{1}{8}$$

$$P(x=1) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$P(x=2) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$P(x=3) = \frac{1}{8}$$

$$P_0 = P(x=0) = {}^3C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

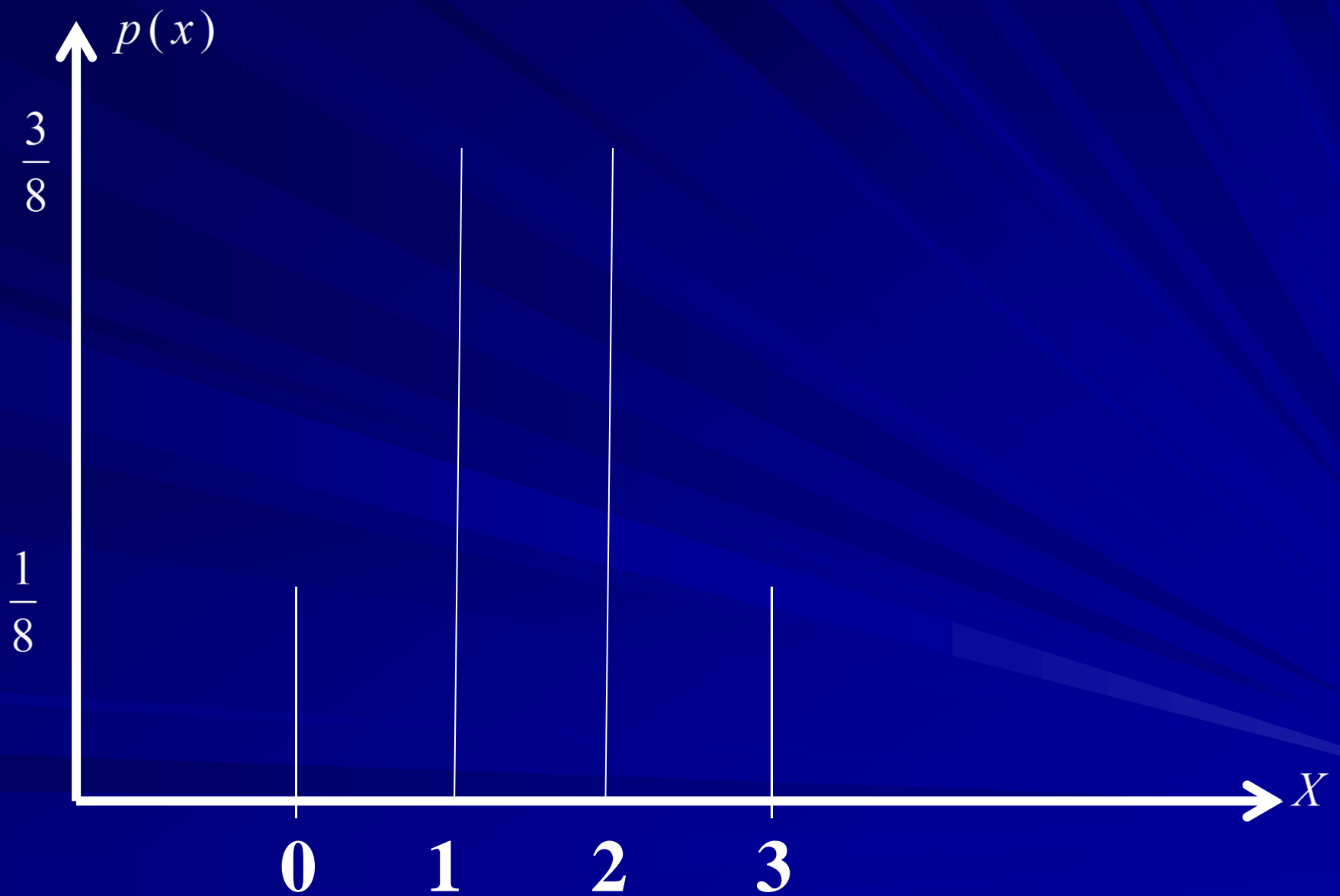
$$P_1 = P(x=1) = {}^3C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P_2 = P(x=2) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P_3 = P(x=3) = {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
F (X)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{8}{8}$

Probability Graph



The expected number of boy

$$E(X) = \frac{1}{8} \times 0 + \frac{3}{8} \times 1 + \frac{3}{8} \times 2 + \frac{1}{8} \times 3 = \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = \frac{1}{8} \times 0^2 + \frac{3}{8} \times 1^2 + \frac{3}{8} \times 2^2 + \frac{1}{8} \times 3^2 = \frac{24}{8} = 3$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 3 - \left[\frac{3}{2}\right]^2 = 3 - \frac{9}{4} = \frac{3}{4} \end{aligned}$$

$$\text{Standard deviation, } \sigma = \sqrt{\text{Var}(X)} = \frac{\sqrt{3}}{2}$$

Pg 107 . EX (1) No. 3

Three marbles are draw without replacement from an urn containing 4 red and 6 white marbles. If X is a random variable which denotes the total number of red marbles drawn,

X be the number of red

$$R_x = \{0, 1, 2, 3\} \quad W = \text{White marble} \quad R = \text{Red marble}$$

$$P(W, W, W) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{30} \quad P(R, W, W) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{5}{30}$$

$$P(W, W, R) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{30} \quad P(R, W, R) = \frac{4}{10} \times \frac{6}{9} \times \frac{3}{8} = \frac{3}{30}$$

$$P(W, R, W) = \frac{6}{10} \times \frac{4}{9} \times \frac{5}{8} = \frac{5}{30} \quad P(R, R, W) = \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} = \frac{3}{30}$$

$$P(W, R, R) = \frac{6}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{3}{30} \quad P(R, R, R) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{30}$$

$$P_0 = P(x=0) = \frac{5}{30}$$

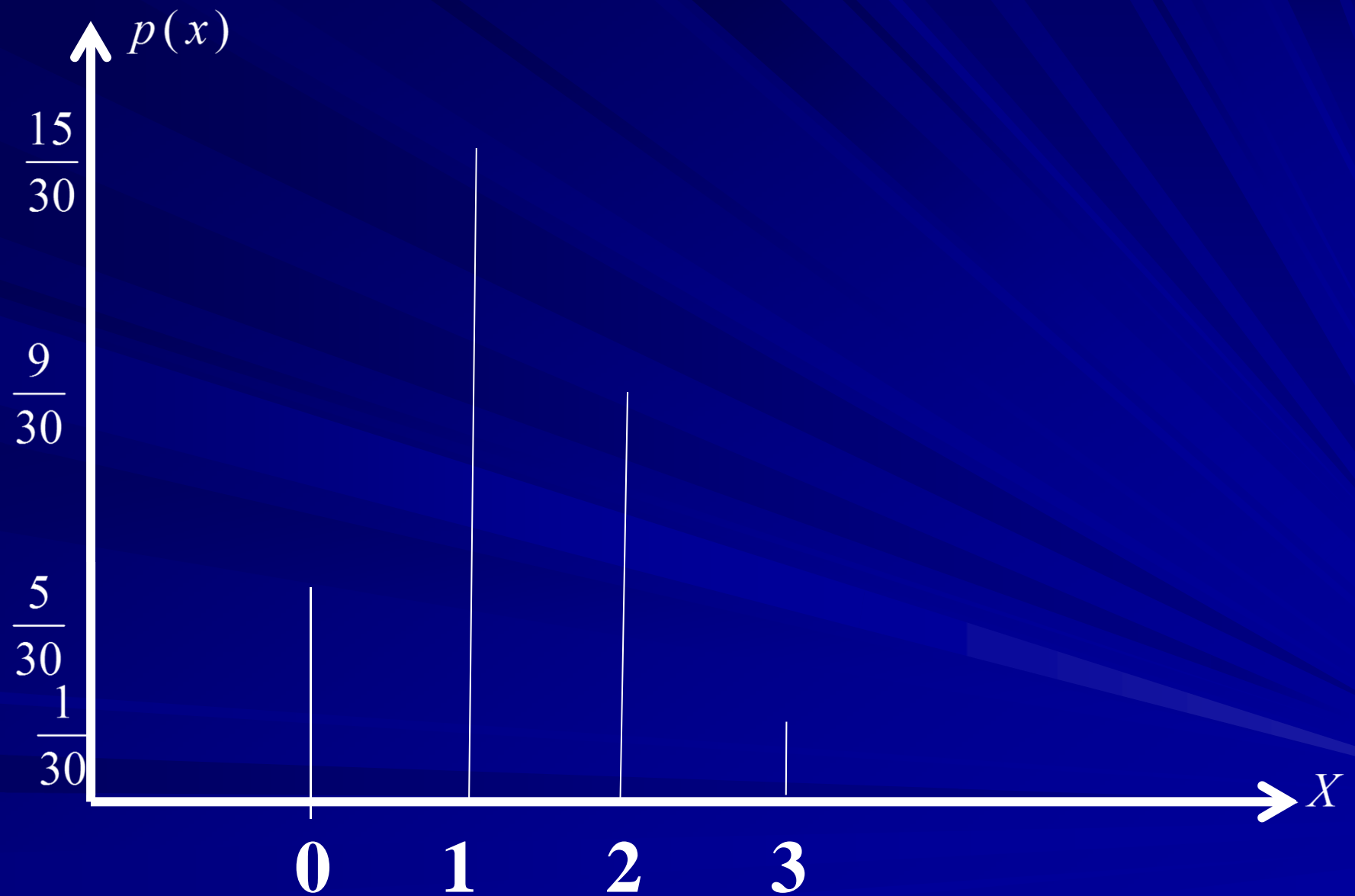
$$P_1 = P(x=1) = \frac{5}{30} + \frac{5}{30} + \frac{5}{30} = \frac{15}{30}$$

$$P_2 = P(x=2) = \frac{3}{30} + \frac{3}{30} + \frac{3}{30} = \frac{9}{30}$$

$$P_3 = P(x=3) = \frac{1}{30}$$

X	0	1	2	3
P(X)	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$
F (X)	$\frac{5}{30}$	$\frac{20}{30}$	$\frac{29}{30}$	$\frac{30}{30}$

Probability Graph



The expected number of red marbles

$$E(X) = \frac{5}{30} \times 0 + \frac{15}{30} \times 1 + \frac{9}{30} \times 2 + \frac{1}{30} \times 3 = \frac{36}{30} = \frac{6}{5}$$

$$E(X^2) = \frac{5}{30} \times 0^2 + \frac{15}{30} \times 1^2 + \frac{9}{30} \times 2^2 + \frac{1}{30} \times 3^2 = \frac{60}{30} = 2$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 2 - \left[\frac{6}{5}\right]^2 = 2 - \frac{36}{25} = \frac{14}{25} \end{aligned}$$

$$\text{Standard deviation, } \sigma = \sqrt{\text{Var}(X)} = \frac{\sqrt{14}}{5}$$

Pg 110 . EX (2) No. 1

If a man purchases a raffle ticket, he can win a first prize of \$ 5000 or a second prize of \$ 2000 with probability 0.001 and 0.003. What should be a fair price to pay for the ticket?

Fair price to pay for the ticket

$$= (0.001) \times (5000) + (0.003) \times (2000) = 11\$$$

Pg 110 . EX (2) No. 2

In a given business venture a man can make a profit of \$ 300 with probability 0.6 or take a loss of \$ 100 with probability 0.4. Determine his expectation.

Expectation of this man

$$= (0.6) \times (300) + (0.4) \times (-100) = 140\$$$

Pg 110 . EX (2) No. 4

A bag contains two white balls and three black balls. Four persons A, B, C, D in order named each draws one ball and does not replace. The first to draw a white ball receives \$ 10. Determine their expectations.

$$E (A) = ?$$

$$E (B) = ?$$

$$E (C) = ?$$

$$E (D) = ?$$

Pg 108 . EX (1) No. 5

A draw contains 8 brown socks and 4 blue socks. A sock is taken from the drawer at random its colour is note and it is then replaced. This procedure is performed twice more. If X is the random variable “the number of brown socks taken”, find the probability distribution of X .

X is the random variable “the number of brown socks taken”

N be the number draw. $N > 2$

$$R_x = \{0, 1, 2, 3, \dots, N\}$$

$$P_0 = P(x=0) = {}^N C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^N = \left(\frac{1}{3}\right)^N$$

$$P_1 = P(x=1) = {}^N C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^{N-1} = N \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^{N-1}$$

$$P_N = P(x=N) = {}^N C_N \left(\frac{2}{3}\right)^N \left(\frac{1}{3}\right)^0 = \left(\frac{2}{3}\right)^N$$

X	0	1	-	-	-	N
P(X)	${}^N C_0 \left(\frac{1}{3}\right)^N$	${}^N C_1 \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{N-1}$				${}^N C_N \left(\frac{2}{3}\right)^N$

Binomial Distribution

N = number of trials

p = probability of success in any single trial of N trials.

$q = 1 - P$ = probability of failure in any single trial of N trials.

X = number of success in N trials.

$N - X$ = number of failure in N trials.

$$P(X=x) = {}^N C_x p^x q^{N-x}, \text{ where } X=0,1,2,\dots, N$$

This discrete probability distribution is often called the binomial distribution since for $X=0, 1, 2, \dots, N$ it corresponds to successive terms in the binomial expansion

$$(p+q)^N = {}^N C_0 p^0 q^{N-0} + {}^N C_1 p^1 q^{N-1} + \dots + {}^N C_N p^N q^{N-N}$$

Some Properties Binomial Distribution

$$\mu = E(X) = Np$$

$$\text{Var}(X) = Npq$$

$$\text{Standard deviation, } \sigma = \sqrt{\text{Var}(X)} = \sqrt{Npq}$$

$$E(N - X) = Nq$$

$$\text{Var}(N - X) = Npq$$

An ordinary die is thrown seven times. Find the probability of (i)obtaining exactly three six.

(ii)obtain at most three six (iii) obtain exactly three

not six(iv) obtain more than five time not six

(v) expectation of getting six (vi) standard deviation

N = the numbers of thrown the die

p = the probability of getting six in any single thrown

q = the probability of not getting six in any single thrown

X = the numbers of times of getting six in N trial

N-X =the numbers of times of not getting six in N trial

$$N = 7 \qquad p = \frac{1}{6} \qquad q = \frac{5}{6}$$

$$N = 7 \quad p = \frac{1}{6} \quad q = \frac{5}{6}$$

(i) obtain exactly three six

$$\mathbf{X = 3} \quad , \quad \mathbf{N - X = 4}$$

$$P(X = x) = {}^N C_x p^x q^{N-x}$$

$$P(X = 3) = {}^7 C_3 p^3 q^4 = {}^7 C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

$$= 35 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

(ii) obtain at most three six

$$X \leq 3$$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= {}^7C_0 p^0 q^7 + {}^7C_1 p^1 q^6 + {}^7C_2 p^2 q^5 + {}^7C_3 p^3 q^4$$

$$= 1 \times \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^6 + \frac{7 \times 6}{2 \times 1} \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5$$

$$+ \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^6 + 21 \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 + 35 \times \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

(iii) obtain exactly three not 6

$$N - X = 3$$

$$7 - X = 3$$

$$X = 4$$

$$P(N - X = 3) = P(X = 4) = {}^7C_4 p^4 q^3 = {}^7C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

$$= 35 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

(iv) obtain more than five time not six

$$N - X > 5$$

$$7 - X > 5$$

$$X < 2$$

$$P(N - X > 5) = P(X < 2) = P(X=0) + P(X=1)$$

$$= {}^7C_0 p^0 q^7 + {}^7C_1 p^1 q^6$$

$$= 1 \times \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^6$$

$$= \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^6$$

(v) expectation of getting six

$$\mu = E(X) = Np$$

$$\mu = E(X) = 7 \times \frac{1}{6}$$

(vi) Standard deviation, $\sigma = \sqrt{\text{Var}(X)} = \sqrt{Npq}$

$$\sigma = \sqrt{7 \times \frac{1}{6} \times \frac{5}{6}}$$

$$\sigma = \frac{\sqrt{35}}{6}$$

The probability that a pen drawn at random from a box of pens is defective is 0.1. If a sample of 6 pens is taken, find the probability that (i) no defective pen (ii) 5 or 6 defective pens (iii) less than 3 defective pen (iv) expectation of defective (v) expectative of non defective (vi) variance

N = the numbers of pens

p = the probability of getting defective pen in any single pen.

q = the probability of getting non defective pen in any single pen.

X = the no. of getting defective pen in 6 pen.

$N - X$ = the no. of getting non defective pen in 6 pen.

$$N = 6$$

$$p = 0.1$$

$$q = 0.9$$

$$(i) \quad X = 0, \quad N - X = 6$$

$$P(X = x) = {}^N C_x p^x q^{N-x}$$

$$P(X = 0) = {}^6 C_0 p^0 q^6 = 1 \times (0.1)^0 (0.9)^6$$

$$= (0.9)^6$$

(ii) $X=5$ or 6

$$P(X=5 \text{ or } 6) = P(X=5) + P(X=6)$$

$$= {}^6C_5 p^5 q^1 + {}^6C_6 p^6 q^0$$

$$= {}^6C_1 p^5 q^1 + {}^6C_0 p^6 q^0$$

$$= 6 \times (0.1)^6 (0.9)^1 + 1 \times (0.1)^6 (0.9)^0$$

$$= 6 \times (0.1)^6 (0.9) + (0.1)^6$$

(iii) $X < 3$

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6C_0 p^0 q^6 + {}^6C_1 p^1 q^5 + {}^6C_2 p^2 q^4$$

$$= 1 \times (0.1)^0 (0.9)^6 + 6 \times (0.1)^1 (0.9)^5 + \frac{6 \times 5}{2 \times 1} \times (0.1)^2 (0.9)^4$$

$$= (0.9)^6 + 6 \times (0.1) (0.9)^5 + 15 \times (0.1)^2 (0.9)^4$$

$$\mu = E(X) = Np$$

$$\mu = E(\text{defective}) = 6 \times 0.1 = 0.6$$

$$\mu = E(\text{non defective}) = 6 \times 0.9 = 5.4$$

$$\text{Variance} = Npq$$

$$= 6 \times (0.1) (0.9)$$

$$= 0.54$$

The probability that a person serves a company A is 0.6. Find the probability that in a randomly selected sample of 8 labourers there are (i) exactly 3 who serve company A. (ii) more than 5 who serve company A. (iii) less than 2 who does not serve company A (iv) expectation of serve company A (v) standard deviation

N = the numbers of labourers

p = the probability of serve company A in any single person

q = the probability of not serve company A in any single person

X = the numbers of person who serve company A in N labourers

$N - X$ = the numbers of person who do not serve company A in N labourers

$$N = 8 \quad p = 0.6 \quad q = 0.4$$

(i) exactly 3 person who serve copany A

$$\mathbf{X = 3}, \mathbf{N - X = 5}$$

$$P(X = x) = {}^N C_x p^x q^{N-x}$$

$$P(X = 3) = {}^8 C_3 p^3 q^5 = {}^8 C_3 (0.6)^3 (0.4)^5$$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} (0.6)^3 (0.4)^5$$

$$= 56 (0.6)^3 (0.4)^5$$

(ii) more than 5 who serve copany A

$$X > 5$$

$$P(X > 5) = P(X=6) + P(X=7) + P(X=8)$$

$$= {}^8C_6 p^6 q^2 + {}^8C_7 p^7 q^1 + {}^8C_8 p^8 q^0$$

$$= {}^8C_2 p^6 q^2 + {}^8C_1 p^7 q^1 + {}^8C_0 p^8 q^0$$

$$= \frac{8 \times 7}{2 \times 1} \times (0.6)^6 (0.4)^2 + 8 \times (0.6)^7 (0.4)^1 + 1 \times (0.6)^8 (0.4)^0$$

$$= 28 \times (0.6)^6 (0.4)^2 + 8 \times (0.6)^7 (0.4) + (0.6)^8$$

(iii) less than 2 who does not serve company A

$$N - X < 2$$

$$8 - X < 2$$

$$X > 6$$

$$P(N - X < 2) = P(X > 6) = P(X = 7) + P(X = 8)$$

$$= {}^8C_7 p^7 q^1 + {}^8C_8 p^8 q^0$$

$$= {}^8C_1 p^7 q^1 + {}^8C_0 p^8 q^0$$

$$= 8 (0.6)^7 (0.4)^1 + 1 \times (0.6)^8 (0.4)^0$$

$$= 8 (0.6)^7 (0.4) + (0.6)^8$$

(iv) expectation of serve copany A

$$\mu = E(X) = Np$$

$$\mu = E(X) = 8 \times 0.6 = 4.8$$

$$\text{Standard deviation, } \sigma = \sqrt{\text{Var}(X)} = \sqrt{Npq}$$

$$\sigma = \sqrt{8 \times 0.6 \times 0.4}$$

Pg 122 , No.14

In a group of people the expected number who wear glasses is 2 and the variance is 1.6. Find the probability that (i) a person chosen at random from the group wear glasses, (ii) 6 people in a group wear glasses.

N = the numbers of people in a group

p = the probability of wear glasses in single person

q = the probability of not wear glasses in single person

X = the no. of persons who wear glasses in a group

$N-X$ = the no. of person who does not wear glasses in a group

$$N = N \quad p = p \quad q = q \quad E(X) = Np = 2 \quad \sigma^2 = Npq = 1.6$$

$$E(X) = Np = 2$$

$$Np = 2 \quad \text{_____} \quad (1)$$

$$\text{Var}(X) = Npq = 1.6$$

$$Npq = 1.6 \quad \text{_____} \quad (2)$$

$$\text{eq}(2) \div \text{eq}(1) \Rightarrow \frac{Npq}{Np} = 0.8$$

$$q = 0.8$$

$$p + q = 1$$

$$p + 0.8 = 1$$

$$p = 0.2$$

The probability that a person chosen at random from the group wear glasses is 0.2

(ii) $X=6$

$$P(X=6) = {}^{10}C_6 p^6 q^4$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 5 \times 4}{6 \times 5 \times 4 \times 3 \times 2 \times 1} \times (0.2)^6 (0.8)^4$$

$$= 210 \times (0.2)^6 (0.8)^4$$

$$(iii) \quad N - X = 3$$

$$7 - X = 3$$

$$X = 4$$

$$P(N - X = 3) = P(X = 4) = {}^7C_4 p^4 q^3 = {}^7C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

$$= 35 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^3$$

A box contain a large number of red and blue tulip bulbs in the ratio $1 : 4$. Bulbs are picked N times with replacement at random from the box,

(i) If $N = 10$, then find the probability that for getting four red bulbs. (ii) If expected numbers of red is 4, then find the probability that for getting five red bulbs. (iii) If standard deviation is 2, then find the probability that for getting four or five blue bulbs.

N = the numbers of draws

p = the probability of getting red in any single draw

q = the probability of getting blue in any single draw

X = the no. of red bulbs in N draws

N-X = the no. of blue bulbs in N draws

$$N = N \quad p = \frac{1}{5} \quad q = \frac{4}{5}$$

A box contains a large number of red and blue tulip bulbs in the ratio 1 : 4. Bulbs are picked N times with replacement at random from the box, if the probability that no blue bulb is 6.4×10^{-5} . Find the probability that (i) for getting at most two red bulbs (ii) less than two blue bulbs.

Pg 120 , No.3

In a multiple choice test there are 10 questions and for each question there is a choice of 4 answers, only one of which is correct. If a student guesses at each of the answers, find the probability that he gets (a) no correct (b) more than 7 correct (c) more than 2 correct.

N = the numbers of questions

p = the probability of correct in any single question

q = the probability of uncorrect in any single question

X = the no. of correct answers in N questions.

$N-X$ = the no. of uncorrect answers in N questions

$$N = 10 \quad p = \frac{1}{4} \quad q = \frac{3}{4}$$

$$N = 10$$

$$p = \frac{1}{4}$$

$$q = \frac{3}{4}$$

(i) no correct

$$P(X=0) = {}^{10}C_0 p^0 q^{10} = \left(\frac{3}{4}\right)^{10}$$

(ii) more than 7 correct

$$P(X > 7) = P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_8 p^8 q^2 + {}^{10}C_9 p^9 q + {}^{10}C_{10} p^{10} q^0$$

$$= 45 \left(\frac{1}{4}\right)^8 \left(\frac{3}{4}\right)^2 + 10 \left(\frac{1}{4}\right)^9 \left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)^{10}$$

(iii) more than 2 correct

$$P(X > 2) = 1 - \{P(X \leq 2)\} = 1 - \{P(X=0) + P(X=1) + P(X=2)\}$$

$$= 1 - \{{}^{10}C_0 p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2 p^2 q^8\}$$

$$= 1 - \left\{ \left(\frac{3}{4}\right)^{10} + 10 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^9 + 45 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 \right\}$$

If the probability that it is the fine day is 0.4, find the expected number of fine days in a week, and the standard deviation. Find the expected number of bad days in a week.

N = the numbers of day in a week

p = the probability of fine day in single day

q = the probability of bad day in single day

X = the numbers of fine days in a week

$N - X$ = the numbers of bad days in a week

$$N = 7 \quad p = 0.4 \quad q = 0.6$$

$$E(X) = Np = 7 \times 0.4 = 2.8$$

$$\text{Standard deviation, } \sigma = \sqrt{\text{Var}(X)} = \sqrt{Npq}$$

$$\sigma = \sqrt{7 \times 0.4 \times 0.6}$$

$$E(N - X) = Nq = 7 \times 0.6 = 4.2$$

The probability that an apple, picked at random from a sack, is bad is 0.05. Find the standard deviation of the number of bad apples in a sample of 15 apples. Find the expected number of bad apple

N = the numbers of sample apples

p = the probability of bad apple in single apple

q = the probability of good apple in single apple

X = the numbers of bad apples in 15 apples

$N - X$ = the numbers of good apples in 15 apples

$$N = 15 \quad p = 0.05 \quad q = 0.95$$

Standard deviation, $\sigma = \sqrt{\text{Var}(X)} = \sqrt{Npq}$

$$\sigma = \sqrt{15 \times 0.05 \times 0.95}$$

$$\mu = E(X) = Np = 15 \times 0.05 = 0.75$$

Poisson Distribution



Poisson Distribution

When n is large and p is small in a binomial distribution.

If x is the numbers of occurrences of random event in an interval of time or space or some volume of matter. The random variable x has a Poisson distribution if and only if the probability distribution is given by

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

When $\lambda = np$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, \dots$$

Poisson Distribution

Examples of events which might follow a poisson distribution distribution:

The number of

- (i) flaws in a given length of material
- (ii) car accidents on a particular stretch of road in one day
- (iii) accidents in a factory in one week
- (iv) telephone calls made to a switchboard in a given time
- (v) insurance claim made to a company in a given time
- (vi) particles emitted by a radioactive source in a given time

A time interval

$$P(x = k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}, \quad k = 0, 1, 2, \dots$$

A given length

$$P(x = k) = \frac{e^{-\lambda l} (\lambda l)^k}{k!}, \quad k = 0, 1, 2, \dots$$

A given region

$$P(x = k) = \frac{e^{-\lambda a} (\lambda a)^k}{k!}, \quad k = 0, 1, 2, \dots$$

A given volume

$$P(x = k) = \frac{e^{-\lambda v} (\lambda v)^k}{k!}, \quad k = 0, 1, 2, \dots$$

Using the Poisson Distribution as an approximation to the Binomial Distribution

A binomial distribution with parameter n and p can be approximated by a poisson distribution, with parameter $\lambda = np$, if n is large (> 50 say) and p is small (< 0.1 say). The approximation get better as $n \rightarrow \infty$ and $p \rightarrow 0$.

$$(p + q)^n \rightarrow e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2!} + \dots \right)$$

When $\lambda = np$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, \dots$$

Find the probability that at least double sixes are obtained when two dice are thrown 90 times. ($e^{-2.5} = 0.082$)

$$\text{Throw two dice, } P(\text{double sixes}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$X \sim \text{Bin}(90, \frac{1}{36}) \text{ and } np = 90 \times \frac{1}{36} = 2.5 \quad , \quad X \sim \text{Poi}(2.5)$$

$$P(x = k) = \frac{e^{-2.5} (2.5)^k}{k!} \quad , \quad k = 0, 1, 2, 3, 4, 5, \dots$$

$$P(x \geq 2) = 1 - \{ P(x=0) + P(x=1) \}$$

$$= 1 - \left(\frac{e^{-2.5} (2.5)^0}{0!} + \frac{e^{-2.5} (2.5)^1}{1!} \right)$$

$$= 1 - e^{-2.5} (1 + 2.5) = 1 - (0.082)(3.5) = 0.713$$

$$Var (X) = E (X^2) - [E (X)]^2$$

$$= (\lambda + \lambda^2) - \lambda^2$$

$$Var (X) = \lambda$$

Mode of Poisson

In general, λ is not an integer, then mode is the integer

$$\lambda - 1 < \text{mod } e < \lambda$$

If the number of bacterial colonies on a petri dish follows a Poisson distribution with average number 2.5 per cm^2 , find the probability that (a) in 1 cm^2 there will be no bacterial colonies (b) in 1 cm^2 there will be more than 4 bacterial colonies (c) in 2 cm^2 there will be 4 bacterial colonies (d) in 4 cm^2 there will be 6 bacterial colonies. ($e^{-2.5} = 0.082$) ($e^{-5} = 0.0067$) ($e^{-10} = 4.5 \times 10^{-5}$)

$$\lambda = 2.5 \text{ per cm}^2$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5, \dots$$

x be the the number of bacterial colonies

$$(a) \quad P(x = 0) = \frac{e^{-2.5} (2.5)^0}{0!} = 0.082$$

$$(a) \ P(x = 0) = \frac{e^{-2.5} (2.5)^0}{0!} = 0.082$$

$$\begin{aligned} (b) \ P(x > 4) &= 1 - \{ P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) \} \\ &= 1 - \left\{ \frac{e^{-2.5} (2.5)^0}{0!} + \frac{e^{-2.5} (2.5)^1}{1!} + \frac{e^{-2.5} (2.5)^2}{2!} + \frac{e^{-2.5} (2.5)^3}{3!} + \frac{e^{-2.5} (2.5)^4}{4!} \right\} \\ &= 1 - e^{-2.5} \left\{ 1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24} \right\} \end{aligned}$$

$$(c) \ \mu = \lambda a = 2.5 \times 2 = 5$$

$$P(x = 4) = \frac{e^{-5} (5)^4}{4!} = \frac{0.067 \times 625}{4!} = 1.745$$

$$(c) \mu = \lambda a = 2.5 \times 2 = 5$$

$$P(x = 4) = \frac{e^{-5} (5)^4}{4!} = \frac{0.067 \times 625}{4!} = 1.745$$

$$(c) \mu = \lambda a = 2.5 \times 4 = 10$$

$$P(x = 6) = \frac{e^{-10} (10)^6}{6!} = \frac{4.5 \times 10^{-5} \times 10^6}{6!} = 0.0139$$

The probability that any telephone subscriber call the swithboard during one hour is 0.1. The telephone station servrices 300 subscribers. What is the probability that 4 subscribers will call the swithboard durind one hour ?What is the probability that at least 3 call in one hour($e^{-3} = 0.0498$)

$$n = 300, p = 0.1 \quad \lambda = np = 300 \times 0.1 = 3$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5$$

x be the the number of subscribers

$$\begin{aligned} P(x = 4) &= \frac{e^{-\lambda} \times \lambda^4}{4!} = \frac{e^{-3} \times 3^4}{4!} = \frac{0.0498 \times 3^4}{4!} \\ &= 1.681 \end{aligned}$$

at least 3 call

$$P(x \geq 3) = 1 - \{P(x=0) + P(x=1) + P(x=2)\}$$

$$= 1 - \left\{ \frac{e^{-3} \times 3^0}{0!} + \frac{e^{-3} \times 3^1}{1!} + \frac{e^{-3} \times 3^2}{2!} \right\}$$

$$= 1 - e^{-3} \{1 + 3 + 4.5\}$$

$$= 1 - (0.0498 \times 8.5)$$

$$= 0.5767$$

Suppose that the probability that a certain type of inoculation take effect is 0.995. What is the probability at most two out of 400 people given the inoculation, find that it has no taken effect ? ($e^{-2} = 0.1315$)

$$n = 400 ,$$

$$P(\text{inoculation taken effect}) = 0.995$$

$$P(\text{inoculation taken no effect}) = 0.005$$

$$\lambda = n p = 400 \times 0.005 = 2$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!} , k = 0, 1, 2, 3, 4, 5$$

x be the the number of people among the 400 from who the inoculation does not effect

$$P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!}$$

$$= e^{-2} [1 + 2 + 2]$$

$$= 0.1353 \times 5$$

$$= 0.6765$$

The expected number of people among the 1000 from who the inoculation does not effect

$$= 1000 \times 0.005 = 5$$

An electric sing is contricted using 1000, 15 watt bulbs. The probability that a new 15 watt bulb will operate for ten hours is given to be 0.995. It all bulbs are new determine the probability that (i) the sing will operate the first ten hours without any bulbs buring out (ii) exactly x bulbs will burn out in the first ten hours (iii) at least two bulbs will burn out in the first ten hours. ($e^{-5} = 0.0067$)

Solution

x be the the number of bulbs burn out in first ten hours

$$P(\text{a new bulb will burn out in first ten hour}) = 1 - 0.995 = 0.005$$

$$\lambda = 1000 \times 0.005 = 5$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5$$

$$(i) P(x = 0) = \frac{e^{-5} \times 5^0}{0!} = 0.0067$$

$$(ii) P(x = x) = \frac{e^{-5} \times 5^x}{x!}$$

$$(iii) P(x \geq 2) = 1 - \{P(x=0) + P(x=1)\}$$

$$= 1 - \left\{ \frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} \right\}$$

$$= 1 - e^{-5} \{1 + 5\}$$

$$= 1 - 0.0067 \times 6$$

$$= 0.9598$$

In a certain region the number of persons who became seriously ill each year eating a certain poisonous plant is a random variable having the Poisson distribution with mean is 2. What is the probability of at most 3 such illnesses in a given year. ($e^{-2} = 0.1315$)

Solution

x be the the number of person who become seriously ill each year from eating a certain poisonous plant

$$\lambda = 2$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5, \dots$$

$$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$\begin{aligned} P(x \leq 3) &= P(x=0) + P(x=1) + P(x=2) + P(x=3) \\ &= \frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!} + \frac{e^{-2} \times 2^3}{3!} \end{aligned}$$

$$= e^{-2} [1 + 2 + 2 + 1.33]$$

$$= 0.1353 \times 6.33$$

$$= 0.8564$$

The probability that a car will have a flat tire while driving over a certain bridge is 0.0002. Find the probability that among 2000 cars driven over the bridge not more than one will have a flat tire. ($e^{-0.4} = 0.67$)

Solution

x be the the number of car which will have flat tire while driving over a certain bridge

$$n = 2000, \quad P(\text{flat tire while driving over}) = 0.0002$$

$$\lambda = 2000 \times 0.0002 = 0.4$$

$$P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5$$

$$P(x \leq 1) = P(x = 0) + P(x = 1)$$

$$P(x \leq 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-0.4} \times (0.4)^0}{0!} + \frac{e^{-0.4} \times (0.4)^1}{1!}$$

$$= e^{-0.4} [1 + 0.4]$$

$$= 0.67 \times 1.4$$

$$= 0.938$$

The average car will have a flat tire while driving over a certain bridge is 0.4 per day. Find the expected number of the day out of 100 days when there will be (i) no flat tire (ii) nor more than one flat tire (iii) between 2 and 5 flat tire.

$$. (e^{-0.4} = 0.67)$$

Solution

x be the the number of car which will have flat tire

$$\lambda = 0.4$$

$$(i) P(x = 0) = \frac{e^{-0.4} (0.4)^k}{0!} = 0.67$$

The expect no. of day which no flat tire

$$= 0.67 \times 100 = 67 \text{ days}$$

$$(ii) P(x \leq 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-0.4} \times (0.4)^0}{0!} + \frac{e^{-0.4} \times (0.4)^1}{1!}$$

$$= e^{-0.4} [1 + 0.4]$$

$$= 0.67 \times 1.4$$

$$= 0.938$$

The expect no. of day which not more one flat tire

$$= 0.938 \times 100 = 93.8 = 94 \text{ days}$$

$$(ii) P(2 < x < 5) = P(x=3) + P(x=4)$$

$$= \frac{e^{-0.4} \times (0.4)^3}{3!} + \frac{e^{-0.4} \times (0.4)^4}{4!}$$

$$= e^{-0.4} [0.0106 + 0.0011]$$

$$= 0.67 \times 0.0117$$

$$= 0.0078$$

The expect no. of day which not more one flat tire

$$= 0.0078 \times 100 = 0.78 = 1 \text{ day}$$

Splices in a certain recording tape occur at random, but on the average of one per 2000 feet. Assume a Poission distribution what is the probability that a 5000 feet root of tape has (i) no splices? (ii) at most two splices ? (iii) at least two splices ?.
($e^{-2.5} = 0.082$)

Solution

x be the the number of splices in the given length 5000ft

$$\lambda = \frac{1}{2000} = 0.0005 \quad l = 5000$$

$$\mu = \lambda l = 0.0005 \times 5000 = 2.5$$

$$P(x = k) = \frac{e^{-\lambda l} (\lambda l)^k}{k!}, \quad k = 0, 1, 2, 3, 4, \dots$$

$$(i) P(x=0) = \frac{e^{-2.5} \times (2.5)^0}{0!} = 0.082$$

$$(ii) P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{e^{-2.5} \times (2.5)^0}{0!} + \frac{e^{-2.5} \times (2.5)^1}{1!} + \frac{e^{-2.5} \times (2.5)^2}{2!}$$

$$= e^{-2.5} [1 + 2.5 + 3.125]$$

$$= 0.082 \times 6.625$$

$$= 0.54325$$

$$(iii) P(x \geq 2) = 1 - \{ P(x=0) + P(x=1) \}$$

$$= 1 - \left[\frac{e^{-2.5} (2.5)^0}{0!} + \frac{e^{-2.5} (2.5)^1}{1!} \right]$$

$$= 1 - e^{-2.5} [1 + 2.5]$$

$$= 1 - 0.082 \times 2.5$$

$$= 0.731$$

Flaws in the painting of target sheet of metal occurs at random. On the average of one in each section of area 10 square feet. What is the probability that a 5 by 8 will have (i) no flaws ? (ii) at most one flaws ? ($e^{-4} = 0.0183$)

Solution

x be the the number of flaws in the given area 40 sq ft

$$\lambda = \frac{1}{10} = 0.1 \quad a = 5 \times 8 = 40$$

$$\mu = \lambda a = 0.1 \times 40 = 4$$

$$P(x = k) = \frac{e^{-\lambda a} (\lambda a)^k}{k!}, \quad k = 0, 1, 2, 3, \dots$$

$$(i) P(x=0) = \frac{e^{-4} \times (4)^0}{0!} = 0.0183$$

$$(ii) P(x \leq 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-4} \times (4)^0}{0!} + \frac{e^{-4} \times (4)^1}{1!}$$

$$= e^{-4} \times [1 + 4]$$

$$= 0.0183 \times 5$$

$$= 0.0915$$

Failure of electron taken airborne applications have been found to follow clearly poisson postulates. A receiver with sixteen tubes suffers. A tube failure on the average of one every 50 hours of operating time. (i) What is the probability of more than one failure on an 8 hours mission ? (ii) What is the expected number of failures in 1000 hours of operation time ? ($e^{-0.16} = 0.8521$)

Solution

x be the the number of tube failure on the given time

$$\lambda = \frac{1}{50} = 0.02 \quad t = 8$$

$$\mu = \lambda t = 0.02 \times 8 = 0.16$$

$$P(x = k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5$$

$$(i) P(x > 1) = 1 - \{ P(x=0) + P(x=1) \}$$

$$= 1 - \left[\frac{e^{-0.16} (0.16)^0}{0!} + \frac{e^{-0.16} (0.16)^1}{1!} \right]$$

$$= 1 - e^{-0.16} [1 + 0.16]$$

$$= 1 - 0.8521 \times 1.16$$

$$= 0.0116$$

$$(ii) \lambda = \frac{1}{50} = 0.02, \quad t = 1000$$

$$\mu = \lambda t = 0.02 \times 1000 = 20$$

Expected number of failure in 1000 hours is 20

A book containing 750 pages has 500 misprints. Assume that the misprints occur at random, find the probability that a particular page contain (i) no misprint (ii) exactly 4 misprints (iii) more than 2 misprints. ($e^{-0.67} = 0.512$)

Solution

x be the the number of misprints in a particular page

$$\lambda = \frac{500}{750} = 0.67$$

$$P(x = k) = \frac{e^{-\lambda} (\lambda)^k}{k!}, \quad k = 0, 1, 2, 3, 4, 5$$