PERMUTATION AND COMBINATION

Factorial Notation

$$n! = n\times(n-1)\times(n-2)\times\ldots\ldots\times3\times2\times1$$

$$7! = 7\times6\times5\times4\times3\times2\times1$$

$$(n+2)! = (n+2)\times(n+1)\times n\times\ldots\ldots\times3\times2\times1$$

$$10! = 10\times9\times8\times7!$$

$$(n+2)! = (n+2)\times(n+1)\times n\times(n-1)!$$

$$(n-2)! = (n-2)\times(n-3)\times(n-4)!$$

$$8\times 7! = 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1 \quad = 8!$$

$$12\times 11! = 12!$$

$$15\times 14\times 13! = 15!$$

$$(n+1)\,n! = (n+1)!$$

$$r\times (r-1)! = r!$$

$$(n-r+1)(n-r)! = (n-r+1)!$$

$$\boxed{Note\,,\quad 0! = 1}$$

The Symbol ${}^{n}p_{r}$

The symbol ${}^{n}P_{r}$ usually denotes the numbers of permutations of n things taken r at a time.

$${}^{n}p_{r} = n(n-1)(n-2)\ldots\ldots(n-r+1)$$

$${}^{10}p_{3} = 10\times9\times8$$

$${}^{n+2}p_{4} = (n+2)(n+1)n(n-1)$$

$${}^{n-2}p_{3} = (n-2)(n-3)(n-4)$$

$n \geq r$, n and r are always positive integer

$$^{n}p_{n} = n(n-1)(n-2).......(n-n+1) = n!$$

$$^{10}P_{3} = \frac{10!}{(10-3)!} = \frac{10\times 9\times 8\times 7!}{7!}$$

$$^{n}p_{r} = \frac{n!}{(n-r)!}$$

$$^{n+2}p_{3} = \frac{(n+2)!}{(n+2-3)!} = \frac{(n+2)!}{(n-1)!}$$

$$^{7}p_{r} = \frac{7!}{(7-r)!}$$

If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$, find n .

$$^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$$

$$\frac{(2n+1)!}{(2n+1-n+1)!} : \frac{(2n-1)!}{(2n-1-n)!} = 3:5$$

$$\frac{(2n+1)!}{(n+2)!} : \frac{(2n-1)!}{(n-1)!} = 3:5$$

$$\frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\frac{(2n+1)(2n)(2n-1)!}{(n+2)(n+1)n(n-1)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\frac{(2n+1)(2n)(2n-1)!}{(n+2)(n+1)n(n-1)!}\times\frac{(n-1)!}{(2n-1)!}=\frac{3}{5}$$

$$\frac{(2n+1)2}{(n+2)(n+1)}=\frac{3}{5}$$

$$\frac{4n+2}{n^2+3n+2}=\frac{3}{5}$$

$$20n+10=3n^2+9n+6$$

$$3n^2-11n-4=0$$

$$(n-4)(3n+1)=0$$

$$n=4 \quad or \quad n=-\frac{1}{3}(inpossible)$$

Pg.25, No.2.

If the number of permutations of 6 things taken r at a time is 360, find r.

the number of permutations of 6 things taken r at a time = 360

$$^6P_r = 360$$

$$\frac{6!}{(6-r)!} = 360$$

$$\frac{6!}{360} = (6-r)!$$

$$\frac{6!}{(6-r)!}=360$$

$$2=(6-r)!$$

$$2!=(6-r)!$$

$$2=(6-r)$$

$$r=4$$

The Symbol $^{n}C_{r}$

The symbol $^{n}C_{r}$ usually denotes the numbers of combinations of n things taken r at a time.

$$^{n}C_{r}=\frac{^{n}P_{r}}{r!}$$

$$^{n}C_{r}=\frac{n(n-1)(n-2)......(n-r+1)}{r!}$$

$$^{n}C_{r}=\frac{n!}{(n-r)!\,r!}$$

$$ {}^{7}C_{3} = \frac{7 \times 6 \times 5}{3!} $$

$$ {}^{n+1}C_{4} = \frac{(n+1)n(n-1)(n-2)}{4!} $$

$$ {}^{n-1}C_{r} = \frac{(n-1)!}{(n-1-r)!\,r!} $$

$$ {}^{n-3}C_{r-4} = \frac{(n-3)!}{(n-3-r+4)!\,(r-4)!} $$

$$ = \frac{(n-3)!}{(n-r+1)!\,(r-4)!} $$

Theorem 1; $^{n}C_{r} = {}^{n}C_{n-r}$

$$e.g,\ {}^{20}C_{15} = {}^{20}C_{5}$$

$$^{n}C_{n-3} = {}^{n}C_{n-n+3} = {}^{n}C_{3}$$

Theorem 2; $^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

$$e.g,\ {}^{20}C_{6} + {}^{20}C_{5} = {}^{21}C_{6}$$

$$e.g,\ {}^{n-1}C_{r-2} + {}^{n-1}C_{r-1} = {}^{n}C_{r-1}$$

$$If\ \ {}^{n}P_{r} = 720\ and\ {}^{n}C_{r} = 120\ find\ r.$$

$${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$$

$$120 = \frac{720}{r!}$$

$$r! = 6$$

$$r! = 3!$$

$$r = 3$$

If ${}^{n}P_{r} = 720$ and ${}^{n}C_{r} = 120$ find $0! \times {}^{n}C_{3}$.

$${}^{n}P_{3} = 720$$

$$n(n-1)(n-2) = 10 \times 9 \times 8$$

$$n = 10$$

$$0! \times {}^{n}C_{3} = 1 \times {}^{10}C_{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

If ${}^{n}P_{r}=840$, ${}^{n}C_{r}=35$, find r .

$$ {}^{n}C_{r}=\frac{{}^{n}P_{r}}{r!} $$

$$ 35=\frac{840}{r!} $$

$$ r!=\frac{840}{35} $$

$$ r!=24 $$

$$ r!=4! $$

$$ r=4 $$

If $^{20}C_{r+4} = {}^{20}C_{2r-4}$, find r.

$$^{20}C_{r+4} = {}^{20}C_{2r-4}$$

$$\frac{20!}{(20-r-4)!(r+4)!} = \frac{20!}{(20-2r+4)!(2r-4)!}$$

$$\frac{1}{(16-r)!(r+4)!} = \frac{1}{(24-2r)!(2r-4)!}$$

$$(16-r)!(r+4)! = (24-2r)!(2r-4)!$$

$$(16-r)! = (24-2r)! \; and \;\; (r+4)! = (2r-4)! \rightarrow eq(1)$$

$$(or)$$

$$(16-r)! = (2r-4)! \; and \;\; (r+4)! = (24-2r)! \rightarrow eq(2)$$

$$(16-r)! = (24-2r)! \; and \; (r+4)! = (2r-4)! \rightarrow eq(1)$$

$$(or)$$

$$(16-r)! = (2r-4)! \; and \; (r+4)! = (24-2r)! \rightarrow eq(2)$$

$$From \; eq(1) \quad 16-r = 24-2r$$

$$r = 8$$

$$(or)$$

$$From \quad eq\,(2) \quad 16-r = 2r-4$$

$$r = \frac{20}{3} (impossible)$$

Permutation and Combination

Permutation

If we are given a number of objects, we may arrange them in different ways. How many different orders can the objects be placed ?

Example

There are 5 routes for going from A to B, 3 routes for going from B to C and 7 routes for going from C to D. Find in how many different ways can a person go from A to D.

The number of different ways for going from A to D is

$$5\times 3\times 7 = 105 \quad ways$$

$$First\ ways \times Second\ ways \times Third\ ways$$

Three digits number formed from 1 to 9 digits

May be repeated

$$9 \times 9 \times 9 = 9^3$$

May not be repeated

$$9 \times 8 \times 7 = {}^9P_3$$

The symbol ${}^nP_r$ usually denotes the numbers of permutations of n things taken r at a time.

Fundamental Principle

If one operation can be performed in m ways, and then second can be performed in n ways, and a third in p ways, and so on

The number of ways of performing all operations in succession will be

$$m \times n \times p \times \ldots\ldots$$

Permutation in which the Quantities may be Repeated

The number of permutation of n things taken r at a time when each thing may occur any number of times is n^r

$$n \times n \times \ldots\ldots \times n \, (r \; times) = n^r$$

Pg.18, No.1.

There are three picture nails on a wall and seven pictures to choose from. In how many different ways can pictures be hung on all the nails?

$$\underline{7} \times \underline{6} \times \underline{5} = {}^{7}P_{3}$$

Pg.18, No.2.

A man has five schools within his reach. In how many ways can he send three of his sons to school, if no two of his sons are to reach in the same school?

Different school $\underset{first\ son}{5} \times \underset{sec\,ond\ son}{4} \times \underset{third\ son}{3} = {}^{5}P_{3}$

Same school $\underset{first\ son}{5} \times \underset{sec\,ond\ son}{5} \times \underset{third\ son}{5} = 5^{3}$

Pg.15, No.8

An exam consists of ten true-or-false questions. Assuming that every question is answered, in how many different ways can a student complete the exam? In how many ways can the exam be completed if a student can leave some questions unanswered because, say, a penalty is assessed for each incorrect answer

The number of way that every questions answered $= 2^{10}$

$\underline{2\times}\ \ \underline{2\times}\ \ \underline{\ 2\ }\ \times\ \ldots\ldots\ldots\times\underline{2}$ **(10th times)**

The number of way that some questions unanswered $= 3^{10}$

$\underline{3\times}\ \ \underline{3\times}\ \ \underline{\ 3\ }\ \times\ \ldots\ldots\ldots\times\underline{3}$ **(10th times)**

Pg.14, No.1

A new state employee is offered a choice of ten basic health plans, three dental plans and two vision care plans. How many different health-care plans are there to choose from if one of the plan is selected from each category?

The number of different ways that

$$= 10 \times 3 \times 2$$

$\underline{10 \times}$ $\underline{3 \times}$ $\underline{2}$

Pg.18, No.2

A warranty identification number for a certain product consists of the letteer followed by a five-digits number. How many possible identification numbers are there if the first digits of the five-digit number must be nonzero?

The number of different ways that

$$= 26 \times 9 \times 10^4$$

$$\underline{26 \times} \quad \underline{9 \times} \quad \underline{10 \times} \quad \underline{10 \times} \quad \underline{10 \times} \quad \underline{10}$$

Pg.14, No.3

A “ lucky dollar ” is one of the nine symbols printed on each reel of a slot machine with three reels. A player receives on a various pay outs whenever one or more “ lucky dollars ” appear in the window of the machine. Find the number of winning combinations for which the machine given a pay off.

The number of different ways that

$$= 9^3 - 8^3$$



Pg.15, No.5

(i) $\underline{9} \times \underline{10} \times \underline{10} \times \underline{10}$ $\qquad$ $\underline{{}^9P_1} \times \underline{10^3}$ ___ ___

(ii) $\underline{9} \times \underline{9} \times \underline{8} \times \underline{7}$ $\qquad$ $\underline{{}^9P_1} \times \underline{{}^9P_3}$ ___ ___

(iii) $\underline{4} \times \underline{10} \times \underline{10} \times \underline{10}$ $\qquad$ $\underline{{}^4P_1}$ ___ $\times \underline{10^3}$ ___

(iv) $\underline{9} \times \underline{10} \times \underline{10} \times \underline{5}$ $\qquad$ $\underline{{}^9P_1}$ ___ $\times \underline{10^2} \times \underline{{}^5P_1}$

Pg.15, No.6

$$\underline{12} \times \underline{11} \times \underline{10} \qquad {}^{12}P_3$$

Pg.15, No.7

$$\left(\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \right) - 10 \qquad 10^4 - 10$$

How many signals may be made with 6 different flags which can be hoisted any number at a time

The number of flags = 6

The number of different ways can be hoisted any number at a time

$$= {}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$$

_ + _ _ + _ _ _ + _ _ _ _ + _ _ _ _ _ + _ _ _ _ _ _

exactly , at least , at most , less than , more than

How many numbers between 3000 and 7000 can be formed by using the digits 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 each not more than once in each number ?

The named of digits are 1, 2 , 3 , 4 , 5 , 6 , 7 and 8.

The number of arrangement ways so

that between 3000 and 7000 = $^4P_1 \times {}^7P_3$

(Not repeated) $\underline{^4P_1}$ / $\underline{\quad}$ $^7P_3$ $\underline{\quad}$ $\underline{\quad}$

(may be repeated) $\underline{^4P_1}$ / $\underline{\quad}$ 8^3 $\underline{\quad}$ $\underline{\quad}$

The number of arrangement ways so that between 300 and 60000 each digits not more than once =

$$\underset{{}^{6}P_1}{\_}\ \_\ \overset{{}^{7}P_2}{\_}\ +\ \underset{{}^{8}P_1}{\_}\ \_\ \overset{{}^{7}P_3}{\_}\ \_\ +\ \underset{{}^{5}P_1}{\_}\ \_\ \_\ \overset{{}^{7}P_4}{\_}\ \_$$

$$(or)\ {}^{8}P_4$$

The number of arrangement ways so that between 300 and 60000 each digits may be repeated =

$$\underset{{}^{6}P_1}{\_}\ \overset{8^2}{\_\ \_}\ +\ \_\ \_\ \overset{8^4}{\_}\ \_\ +\ \underset{{}^{5}P_1}{\_}\ \_\ \overset{8^4}{\_\ \_}\ \_$$

The no. of even numbers so that between 100 and 100000 each digits not more than once =

The no. of odd numbers so that between 100 and 100000 each digits may be repeated =

Pg 18. No.4

How many numbers between 300 and 70000 can be formed by using the digits 0 , 2 , 3 , 4 , 5 , 6 , 7 , 8 each not more than once in each number ?

The name of digits are 0 , 2 , 3 , 4 , 5 , 6 , 7 and 8

The number of arrangement ways that between 300 and 70000 each digits not more than once =

_ _ _ + _ _ _ _ + _ _ _ _ _ $\left({}^6P_1 \times {}^7P_2 + {}^7P_1 \times {}^7P_3 + {}^5P_1 \times {}^7P_4 \right)$

The number of arrangement ways that between 300 and 70000 each digits may be repeated =

$$\left({}^6P_1 \times 8^2 + {}^7P_1 \times 8^3 + {}^5P_1 \times 8^4 \right) - 1$$

Pg.18 , No.6.

In how many ways can 3 different copper coins and 3 different silver coins be arranged in a line so that the silver coins may be in the odd place

S C S C S C $\quad {}^{3}P_{3} \times {}^{3}P_{3}$

The no. of ways that silver coins are odd place

$$= {}^{3}p_{3} \times {}^{3}p_{3}$$

The no. of ways that alternately $\quad = {}^{3}p_{3} \times {}^{3}p_{3} + {}^{3}p_{3} \times {}^{3}p_{3}$

S C S C S C + C S C S C S

The no. of ways that silver coins are not adjacent = ? $\quad {}^{3}p_{3} \times {}^{4}p_{3}$

The number of letters = 26

The number of digits = 10

The no. of ways that to contain 3 letters follow by 5 digits =

L L L D D D D D

(not repeated) $\underline{\quad}\ \underline{\quad}\ \underline{\quad}$ / $\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$ $\quad {}^{26}P_3 \quad {}^{10}P_5$

(repeated) $\underline{\quad}\ \underline{\quad}\ \underline{\quad}$ / $\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$ $\quad 26^3 \quad 10^5$

The no. of ways that to contain 4 letters and 4 digits and letters are odd places

L D L D L D L D

(Not repeated) $= {}^{26}P_4 \times {}^{10}P_4$

(Not repeated) $= 26^4 \times 10^4$

The no. of ways that to contain 4 letters and 4 digits and letters are odd places

L D L D L D L D

(Not repeated) $= {}^{26}P_4 \times {}^{10}P_4$

(repeated) $= 26^4 \times 10^4$

The no. of ways that alternately

(Not repeated) $= {}^{26}P_4 \times {}^{10}P_4 + {}^{26}P_4 \times {}^{10}P_4$

(repeated) $= 26^4 \times 10^4 + 26^4 \times 10^4$

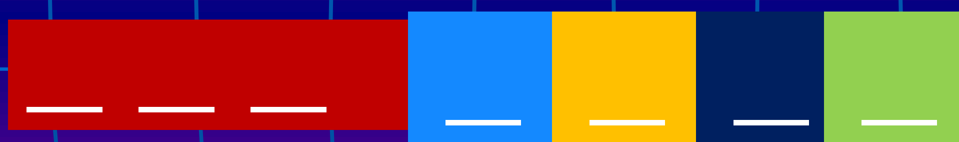
L D L D L D L D + D L D L D L D L

Pg.18 , No.7.

In how many ways can 7 different letters be arranged (i) if 3 of the letters are always all to appear together (ii) if 3 of the letters are never all to appear together ?

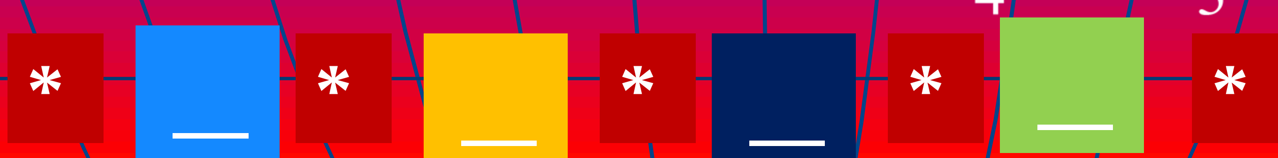
The number of letters = 7

The number of arrangement ways so that 3 of the letters are always all to appear together = $^{5}P_{5} \times {}^{3}P_{3}$



The number of arrangement ways so that 3 of the letters are never all to appear together = $^{7}P_{7} - ({}^{5}P_{5} \times {}^{3}P_{3})$

The number of arrangement ways so that 3 of the letters are separated = $^{4}P_{4} \times {}^{5}P_{3}$

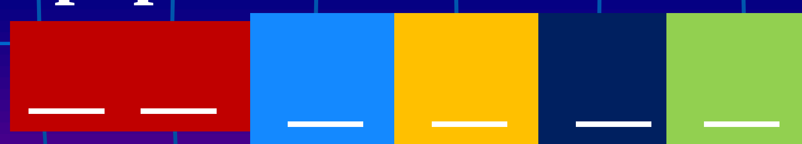


pg.19 , No.9.

Six papers are set in an examination of which two are mathematical. In how many different orders can the papers be arranged, so that the two mathematical papers are not consecutive.

The number of papers = 6

The number of arrangement ways so that two mathematical papers are consecutive = $^5P_5 \times {}^2P_2$



The number of arrangement ways so that two mathematical papers are not consecutive =

$$^6P_6 - ({}^5P_5 \times {}^2P_2)$$

Pg.18 , No.5.

How many new words can be formed by using the letters of the word – UNIVERSAL so that the central letter may be the vowel.

In the word UNIVERSAL ,there are 9 letters

The number of vowels = 4

The number of words can be formed such that central as a vowel = $^4P_1 \times {}^8P_8$

$$\_\_ \;\_\_ \;\_\_ \;\_\_ \;\underset{^4P_1}{\overset{V}{\_\_}} \;\_\_ \;\_\_ \;\_\_ \;\_\_$$

The number of new words can be formed such that central as a vowel = $^4P_1 \times {}^8P_8 - 1$

The number of words can be formed such that begin and end with consonants =

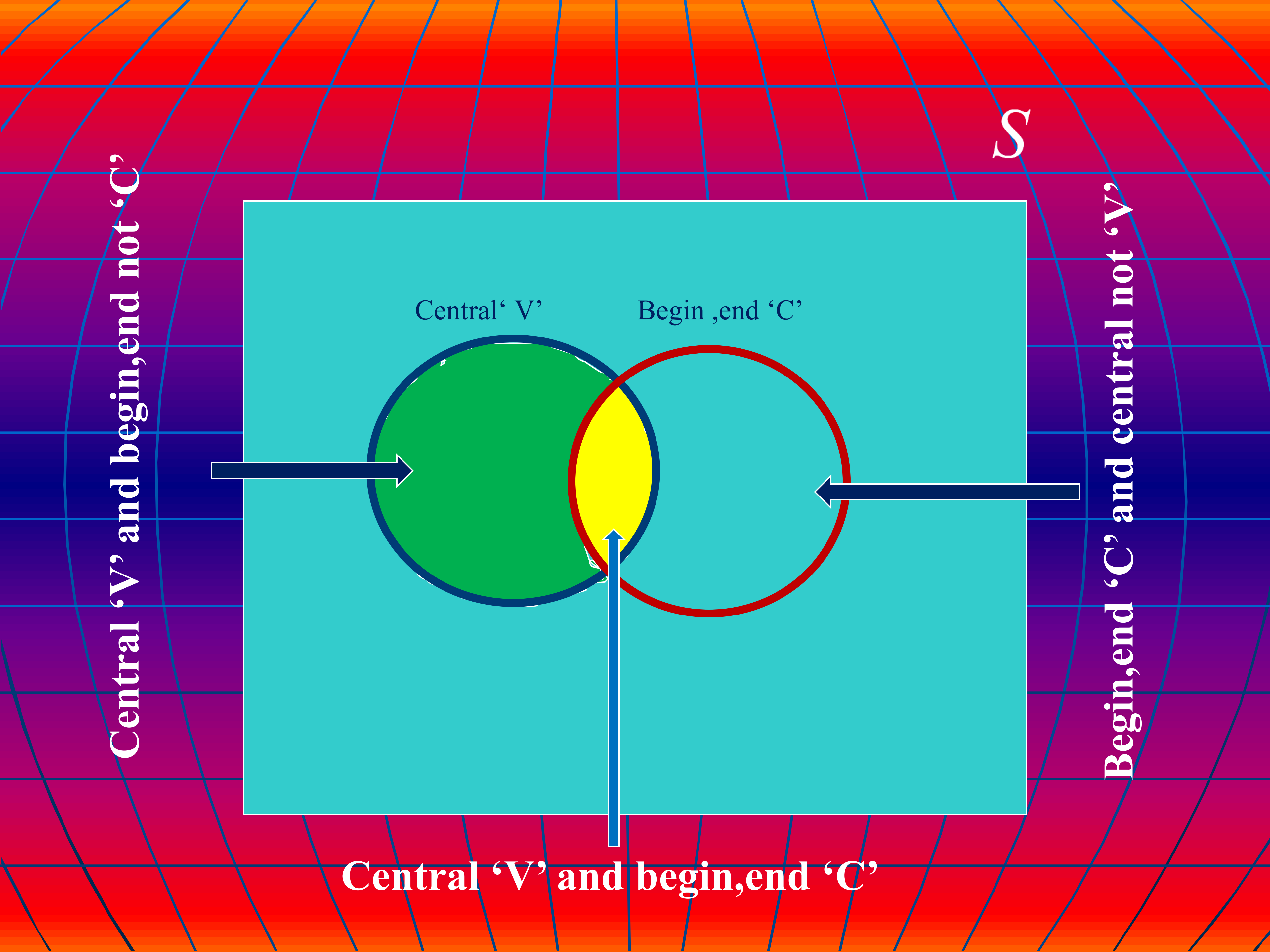
$${}^{5}P_{2} \times {}^{7}P_{7}$$

$$\underline{C}\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \underline{C}$$

The number of words can be formed such that central letter may be vowel and begin and end with consonants =

$${}^{4}P_{1} \times {}^{5}P_{2} \times {}^{6}P_{6}$$

$$\underline{C}\ \_\_\ \_\_\ \_\_\ \underline{V}\ \_\_\ \_\_\ \_\_\ \underline{C}$$



S
Central‘ V’
Begin ,end ‘C’
Central ‘V’ and begin,end not ‘C’
Begin,end ‘C’ and central not ‘V’
Central ‘V’ and begin,end ‘C’

The number of words can be formed such that all vowels are together = ${}^{6}P_{6} \times {}^{4}P_{4}$

V V V V   C C C C C

The number of words can be formed such that all vowels are separated = ${}^{5}P_{5} \times {}^{6}P_{4}$

C C C C C

Pg.19 , No.10.

Show that the number of ways in which 4 girls and 4 boys can be arranged, so that no two girls may be together is 5 ! × 4 ! .

g b g b g b g b g

First, we arrange the 4 boys $= {}^4p_4 = 4!$

The no. of chosen ways for first girl = 5

The no. of chosen ways for second girl = 4

The no. of chosen ways for third girl = 3

The no. of chosen ways for last girl = 2

The no. of required ways = $4! \times 5 \times 4 \times 3 \times 2$

$= 4! \times 5 \times 4 \times 3 \times 2 \times 1 = 4! \times 5!$

Pg.19 , No.11.

In how many orders can five girls and three boys walk through a doorway single file if

(a) there is no restriction ?

(b) the girls walk through before the boys ?

$$(a)\ {}^{8}P_{8} = 8!$$

$$(b)\ {}^{5}P_{5} \times {}^{3}P_{3} = 5! \times 3!$$

GGGG G **B B B**

Pg.19 , No.12.

Three couples have reserved seats in a row for a concret. In how many different ways can they be seated if

(a) there are no restrictions ?

(b) the two members of each couple wish to sit together

$$(a)\ {}^6P_6 = 6!$$

$$(b)\ {}^3P_3 \times {}^2P_2 \times {}^2P_2 \times {}^2P_2 = 3! \times 2! \times 2! \times 2!$$

B G	B G	B G

Pg19 , No.14

In how many ways can 5 different prizes be given to a class of 30 boys, when each boy is eligible all the prizes ?

The number of different prizes = 5

The number of boy = 30

The number of ways for prizes be given $= 30^5$ *ways*

Pg19 , No.15

I have 5 parcels to be sent, and there are three post offices within my reach. In how many ways can I sent them?

The number of different prizes = 5

The number of post offices = 3

The number of required ways = ?

Pg 19, No.16

(a)How many numbers of 4 digits can be formed with the digits 1 , 2 , 3 ?

(b)How many numbers less than 10000 can be formed from the digits 1 , 2 , 3 ?

The numbers of 4 digits can be formed with the digits 1 , 2 , 3 $= 3^4$

The numbers less than 10000 can be formed from the digits 1 , 2 , 3

$$= 3 + 3^2 + 3^3 + 3^4$$

Pg19, No.17.

How many number of 5 digits can be formed with 0 , 1 , 3 , 4 , 5 , 7 and 9 , if each of these digits may be repeated? Of these how many are even and how many are divisible by 5?

The digits are 0 , 1 , 3 , 4 , 5 , 7 , 9

$\underline{{}^6P_1}$ ___ $\underline{7^4}$ ___ ___

(i)The numbers of 5 digits can be formed $= {}^6P_1 \times 7^4$

(ii)The no. of 5 digits even number = ${}^6P_1 \times {}^2p_1 \times 7^3$

$\underline{{}^6P_1}$ ___ $\underline{7^3}$ ___ $\underline{{}^2P_1}$

${}^6P_1$ ___ 7^3 ___ ___ ${}^2P_1$

(iii)The no. of 5 digits number which is divisible by 5 =

(iv)The no. of 5 digits number which is divisible by 25 =

_ _ _ 0 0 + _ _ _ 2 5 + _ _ _ 5 0

${}^6P_1 \times 7^2$ ${}^6P_1 \times 7^2$ ${}^6P_1 \times 7^2$

Consider, the no. of 5 digits number each digits not more than once.

The digits are 0 , 1 , 3 , 4 , 5 , 7 , 9 (not repeated)

(i)The numbers of 5 digits can be formed $= {}^{6}P_{1} \times {}^{6}P_{4}$

(ii)The no. of 5 digits even number = ?

$${}^{6}\mathrm{P}_{4} + {}^{5}\mathrm{P}_{1} \times {}^{5}\mathrm{P}_{3}$$

(iii)The no. of 5 digits number which is divisible by 5 = ?

$${}^{6}P_{4} + {}^{5}P_{1} \times {}^{5}P_{3}$$

(iv)The no. of 5 digits number which is divisible by 25 = ?

Permutation of n things not all different

Let n things be represented by letters, and p of them are alike, q of them are alike, of them are alike; and so on.

The required number of permutations is

$$\frac{n!}{p!\times q!\times r!\times \dots\dots\dots}$$

A A A B B C C C C

A A A B B C C C C

A A A B B C C C C and so on

$$\frac{9!}{3!\times 2!\times 4!}$$

Pg.21,No.1

In the word COMMITTEE, there are 9 letters, 1 C's , 1 O's , 2 M's , 1 I's , 2 T's and 2 E's .

The no. of total arrangement ways $= \dfrac{9!}{2!\times 2!\times 2!}$

The no. of ways that begin and end with consonants

$\underline{C}$ __ __ __ __ __ __ __ $\underline{C}$ $= \dfrac{{}^{5}P_{2}\times 7!}{2!\times 2!\times 2!}$

The no. of ways that central letters may be vowels and begin and end with consonants

$= \dfrac{{}^{4}P_{1}\times {}^{5}P_{2}\times 6!}{2!\times 2!\times 2!}$

$\underline{C}$ __ __ __ $\underline{V}$ __ __ __ $\underline{C}$

The no. of ways that all vowels are always come to appear together = ?

The no. of ways that all vowels are not come to appear together = ?

The no. of ways that all vowels are to be separated = ?

Pg 21, No. 2

In the word INFINTTESIMAL, there are 13 letters, 3 I's , 2 N's , 1 F's , 2 T's , 1 E's , 1 S's , 1 M's 1 A's and 1 L's .

The number of vowels = 5

The number of arrangement ways $= \dfrac{13!}{3!\times 2!\times 2!}$

The number of arrangement ways such that vowel as a central $= \dfrac{{}^{5}P_{1}\times 12!}{3!\times 2!\times 2!}$

The number of arrangement ways such that all vowel always come together

Pg.21, No.3

In the word CONSONANTS, there are 10 letters, 1 C's , 2 O's , 3 N's , 2 S's , 1 A's and 1 T's .

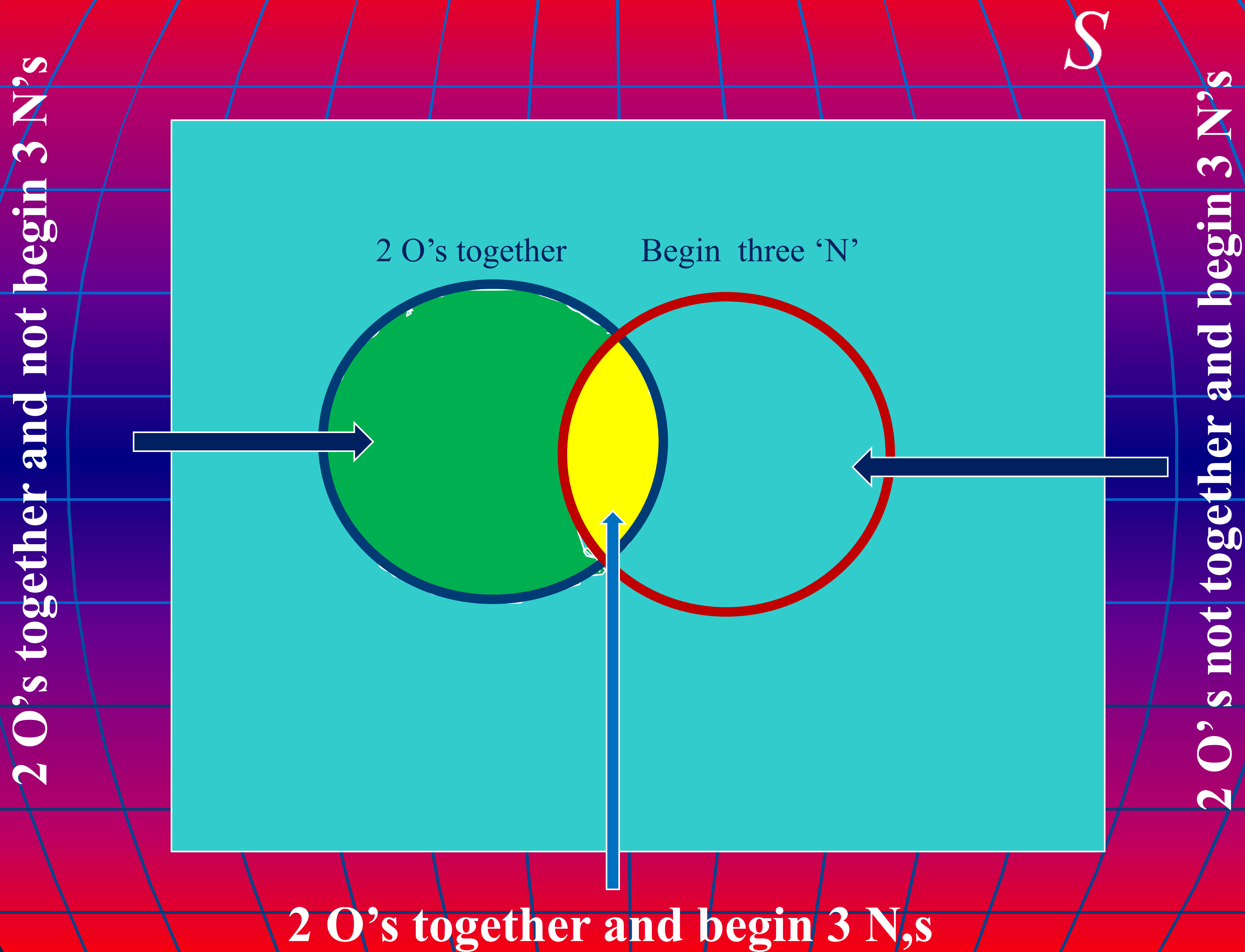
The number of arrangement ways $= \dfrac{10!}{2!\times 2!\times 3!}$

$(\underline{O}\ \underline{O})\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

The number of arrangement ways such that the two O's always come together $= \dfrac{9!}{2!\times 3!}$

The number of arrangement ways such that begin with the three N's

$\underline{N}\ \underline{N}\ \underline{N}\ /\ \_\ \_\ \_\ \_\ \_\ \_\ \_ \quad = \dfrac{7!}{2!\times 2!}$


S
2 O’s together
Begin three ‘N’
2 O’s together and not begin 3 N’s
2 O’ s not together and begin 3 N’s
2 O’s together and begin 3 N,s

The number of arrangement ways such that the two O's always come together and begin with 3 N's $= \frac{6!}{2!}$

$\underline{N}\ \underline{N}\ \underline{N}\ /(\underline{O}\ \underline{O})\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_$

The number of arrangement ways such that the two O's always come together and do not begin with 3 N's

The number of arrangement ways such that the two O's never come together and begin with N's

Pg.21, No.4

The maned of digits are 2 , 2 , 2 , 3 , 3 , 4 , 0 .

The no. of number over 2000000 $= \dfrac{{}^{6}P_{1} \times 6\,!}{3! \times 2!}$

$${}^{6}P_{1} \quad \_\_ \quad \overset{6!}{\_\_} \quad \_\_ \quad \_\_ \quad \_\_ \quad \_\_$$

The no. of even number over 2000000 numbers =

$$= \frac{6\,!}{3! \times 2!} + \frac{{}^{4}P_{1} \times {}^{5}P_{1} \times 5!}{3! \times 2!}$$

The no. of odd number over 2000000 numbers =

$$= \frac{{}^{5}P_{1} \times 5\,!}{3! \times 2!}$$

Pg.21, No.5

In the word ENGINEERING, there are 11 letters, 3 E's , 3 N's , 2 G's , 2 I's and 1 R's .

The number of arrangement ways $= \dfrac{11!}{3!\times 3!\times 2!\times 2!}$

The number of arrangement ways such that the three E's always come together $= \dfrac{9!}{3!\times 2!\times 2!}$

The number of arrangement ways such that begin with the three E's and end N $= \dfrac{7!}{2!\times 2!\times 2!}$

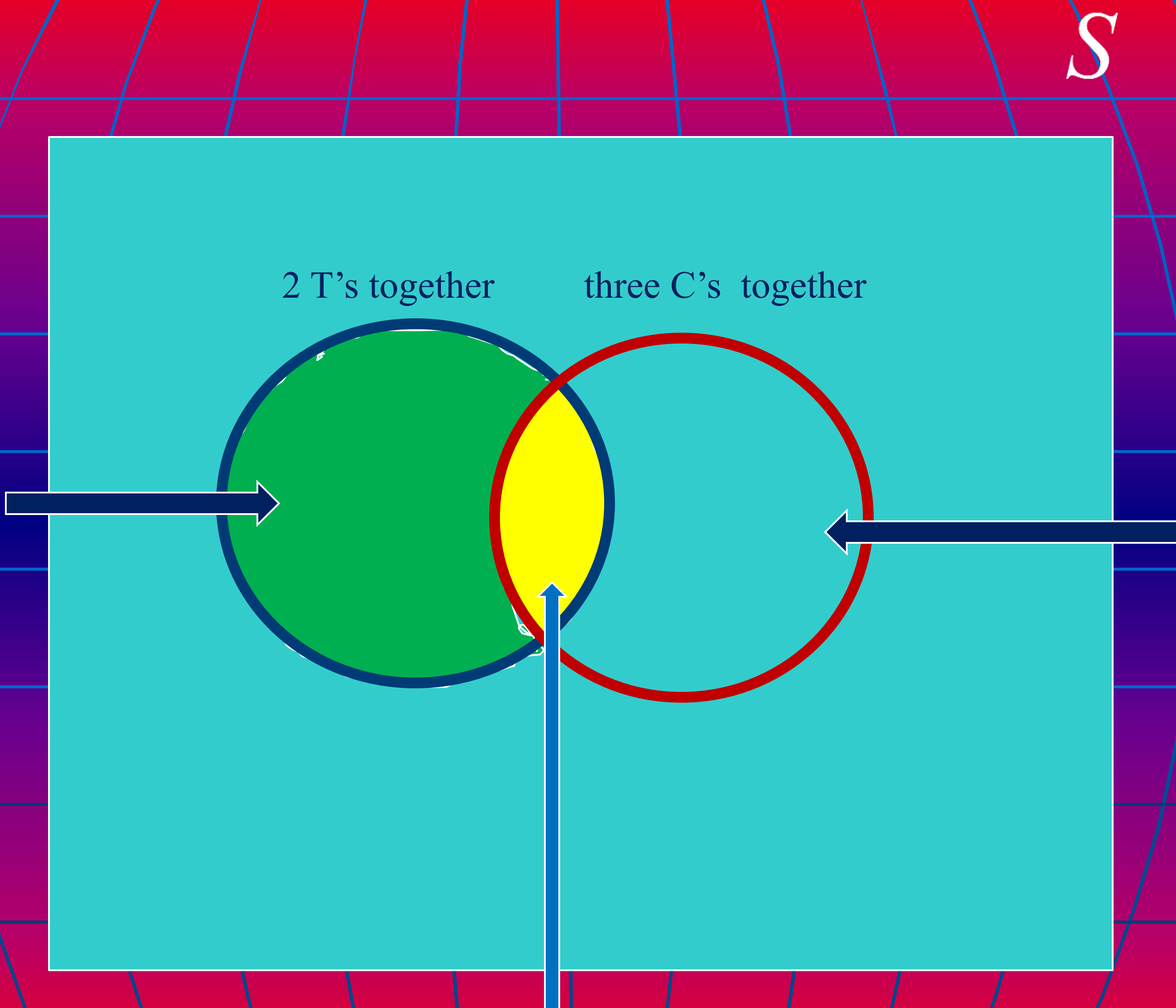
Pg.21, No.6

In the word CHARACTERISTICS, there are 15 letters, 3 C's , 1 H's , 2 A's , 2 R's , 2 T's , 1 E's ,2 I's and 2 S's .

The number of arrangement ways $= \dfrac{15!}{3!\times(2!)^5}$

The number of arrangement ways such that the two R's always come together $= \dfrac{14!}{3!\times(2!)^4}$

The number of arrangement ways such that two R's do not come together $= \dfrac{15!}{3!(2!)^5} - \dfrac{14!}{3!\times(2!)^4}$



2 T’s together and 3 C’s not together

2 T’ s not together and 3 C’s togrther

2 T’s together and 3 C,s together

The number of arrangement ways such that the two T's always come together $= \dfrac{14!}{3!(2!)^4}$

The number of arrangement ways such that two T's come together and 3 C's come together $= \dfrac{12!}{(2!)^4}$

The number of arrangement ways such that two T's come together and 3 C's do not come together

$$= \frac{14!}{3!(2!)^4} - \frac{12!}{(2!)^4}$$

The number of ways that the two T's and 2 C’s are always come together =

The number of ways that the two T's together and 2 C’s together =

Circular Permutation

The circular permutation of n things taken all together

The number of ways in which n different things can be arranged in a round table

$$(n-1)!$$

The number of ways in which n different things can be arranged in a ring

$$\frac{(n-1)!}{2}$$

A B C D E

E A B C D

D E A B C

C D E A B

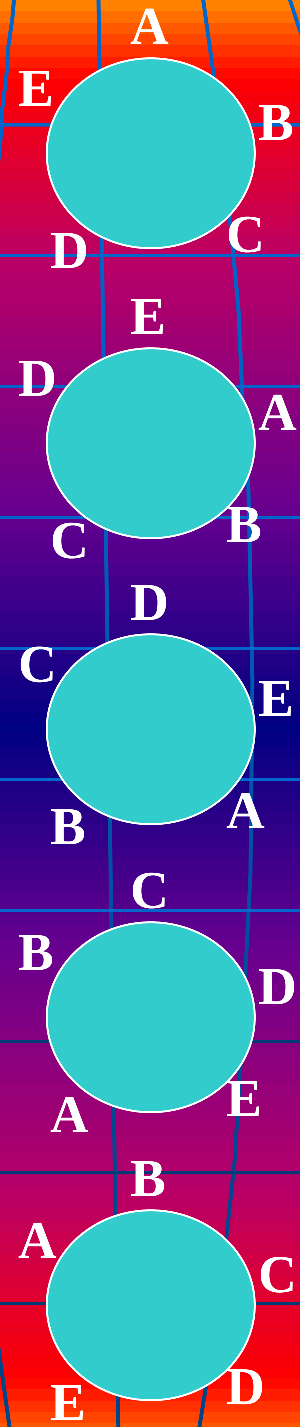
B C D E A

-

-

-

$$^{5}P_{5} = 5! \, ways$$



The number of arrangement ways for a round table is

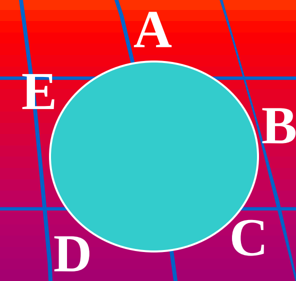
$$\frac{5!}{5} = \frac{5 \times (5-1)!}{5}$$

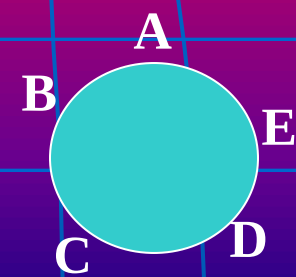
$$\frac{n!}{n} = \frac{n \times (n-1)!}{n}$$

$$= (n-1)!$$

The number of arrangement ways for a round table is

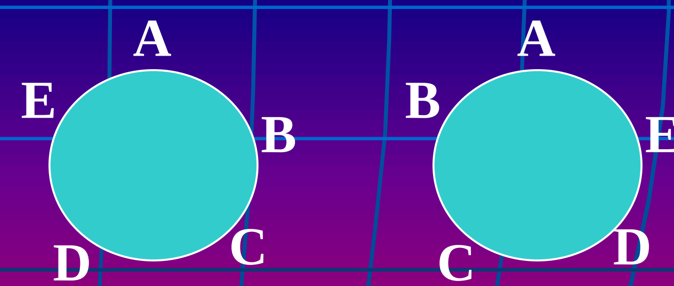
$= (n-1)!$





The number of arrangement ways for a ring is

$$=\frac{(n-1)!}{2}$$



same ways

Pg.21, No.2

The number of persons = 7

The number of ways 7 persons can be seated at a round table, so that 3 of them are never to be separated $= (5-1)! \times 3!$

Pg.22, No.4

The number of keys = 7

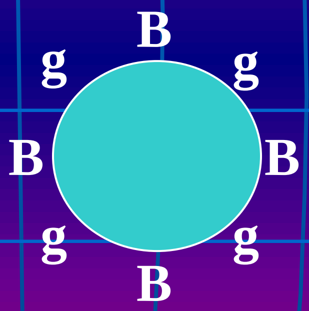
The number of ways 7 different keys can be placed on a keys ring, so that 2 of them are never to be separated $= \dfrac{(6-1)! \times 2!}{2}$

Pg.22, No.5

There are 4 men and 4 women

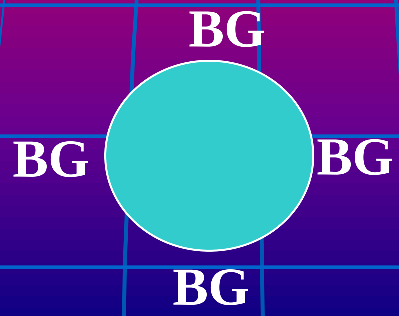
The number of total ways $= (8-1)!$

The no. of ways that each guest is seated between members of the opposite sex $= (4-1)! \times 4!$



There are 4 couples

The no. of ways that the two member of each couple wish to sit together in round table



There are 7 boys and 3 girls

The no. of ways that all girls are separated in round table = ?