PROBABILITY

Probability

$$\text{Probability} = \frac{\text{the number of favourable outcomes}}{\text{the number of possible outcomes}}$$

$$0 \le Pobability \le 1$$

$$P(sample\ \ space) = P(S) = 1$$

$$P(A) = 1 - P(A')$$

*** A and B are independence event**

$$P(A \cap B) = P(A)P(B)$$

*** A and B are mutually exclusive. i.e $A \cap B = \phi$**

$$P(A \cup B) = P(A) + P(B)$$

1 . The number of total items = 12

The number of chosen items = 2

The number of defective items = 4

The number of non defective items= 8

The number of possible outcomes

$$= {}^{12}C_2$$

A = { both items are defective }

8 Non defective	3 Defective
2 1 0	0 1 2

A = { both items are defective }

The number of favorable outcomes for getting both items are defective

$$= {}^{4}C_{2} \times {}^{8}C_{0}$$

P (both items are defective)

$$= \frac{{}^{4}C_{2}}{{}^{12}C_{2}}$$

B = { both items is non defective }

The number of favorable outcomes for getting both items are non defective

$$= {}^{8}C_{2} \times {}^{4}C_{0}$$

P (both items are non defective)

$$= \frac{{}^{8}C_{2}}{{}^{12}C_{2}}$$

2 . The number of red balls = 2

The number of blue balls = 4

The number of white balls = 5

The number of total balls = 11

The number of selected balls = 4

The number of possible outcomes

$$= {}^{11}C_4$$

E = { exactly two white balls }

The number of favorable outcomes for getting exactly two white balls

$$= {}^{5}C_{2} \times {}^{6}C_{2}$$

P (exactly two white balls)

$$= \frac{{}^{5}C_{2} \times {}^{6}C_{2}}{{}^{11}C_{4}}$$

The number of favorable outcomes for getting at least two white balls

$$= {}^{5}C_{2} \times {}^{6}C_{2} + {}^{5}C_{3} \times {}^{6}C_{1} + {}^{5}C_{4} \times {}^{6}C_{0}$$

P (at least two white balls)

$$= \frac{}{{}^{11}C_{4}}$$

The number of favorable outcomes for getting at most two blue balls

$$= {}^4C_2 \times {}^7C_2 + {}^4C_1 \times {}^7C_3 + {}^4C_0 \times {}^7C_4$$

P (at most two blue balls)

$$= \frac{\quad}{{}^{11}C_4}$$

The number of favorable outcomes for getting particular 1 red ball always include and at most two blue balls

$$= {}^4C_2 \times {}^6C_1 + {}^4C_1 \times {}^6C_2$$

P (Particular 1 red ball always include and at most two blue balls)

$$= \frac{}{{}^{11}C_4}$$

3 . The number of boys = 5

The number of teachers = 3

The number of persons in a committee = 6

The number of possible outcomes

$$= {}^{8}C_{6}$$

The number of favorable outcomes for getting majority of boys

$$= {}^{5}C_{5} \times {}^{3}C_{1} + {}^{5}C_{4} \times {}^{3}C_{2}$$

P (majority of boys)

$$= \frac{}{{}^{8}C_{6}}$$

5 . The number of men = 10

The number of men in a group = 6

The number of possible outcomes

$$= {}^{10}C_6$$

The number of favorable outcome that a certain 2 men are included in a group

$$= {}^{10-2}C_{6-2}$$

P (a certain two men are included)

$$= \frac{\qquad}{{}^{10}C_{6}}$$

6 . The number of red balls = 3

The number of black balls = 5

The number of yellow balls = 4

The number of total balls = 12

The number of chosen balls = 3

The number of possible outcomes $= {}^{12}C_3$

The number of favorable outcomes for

getting all yellow $= {}^{4}C_3 \times {}^{8}C_0$

The number of possible outcomes $= {}^{12}C_3$

The number of favorable outcomes

forgetting all yellow $= {}^{4}C_3 \times {}^{8}C_0$

$$P(all\ yellow) =$$

The number of favorable outcomes for

getting one of each color

$$= {}^{3}C_1 \times {}^{5}C_1 \times {}^{4}C_1$$

$$\mathrm{P(one\ of\ each\ colour\ )} =$$

$4\ W\ ,2\ B$

$2\ W\ ,5\ B$

8 .The number of possible outcomes

$$= {}^{6}C_{1} \times {}^{7}C_{1}$$

8 .The number of possible outcomes

$$= {}^{6}C_{1} \times {}^{7}C_{1}$$

The number of favorable outcomes for getting both are white

$$= {}^{4}C_{1} \times {}^{2}C_{0} \times {}^{2}C_{1} \times {}^{5}C_{0}$$

P (both are white) =

The number of favorable outcomes for getting both are black

$$= {}^4C_0 \times {}^2C_1 \times {}^2C_0 \times {}^5C_1$$

P (both are black) =

The number of favorable outcomes for getting 1 white and 1 black

$$= {}^4C_1 \times {}^2C_0 \times {}^2C_0 \times {}^5C_1 + {}^4C_0 \times {}^2C_1 \times {}^2C_1 \times {}^5C_0$$

P (1 white and 1 black) =

4.

$2W, 4R$

$4W, 2R$

4 .The number of possible outcomes

$$= {}^6C_1 \times {}^6C_1$$

4 .The number of possible outcomes

$$= {}^{6}C_{1} \times {}^{6}C_{1}$$

The number of favorable outcomes for getting both are red

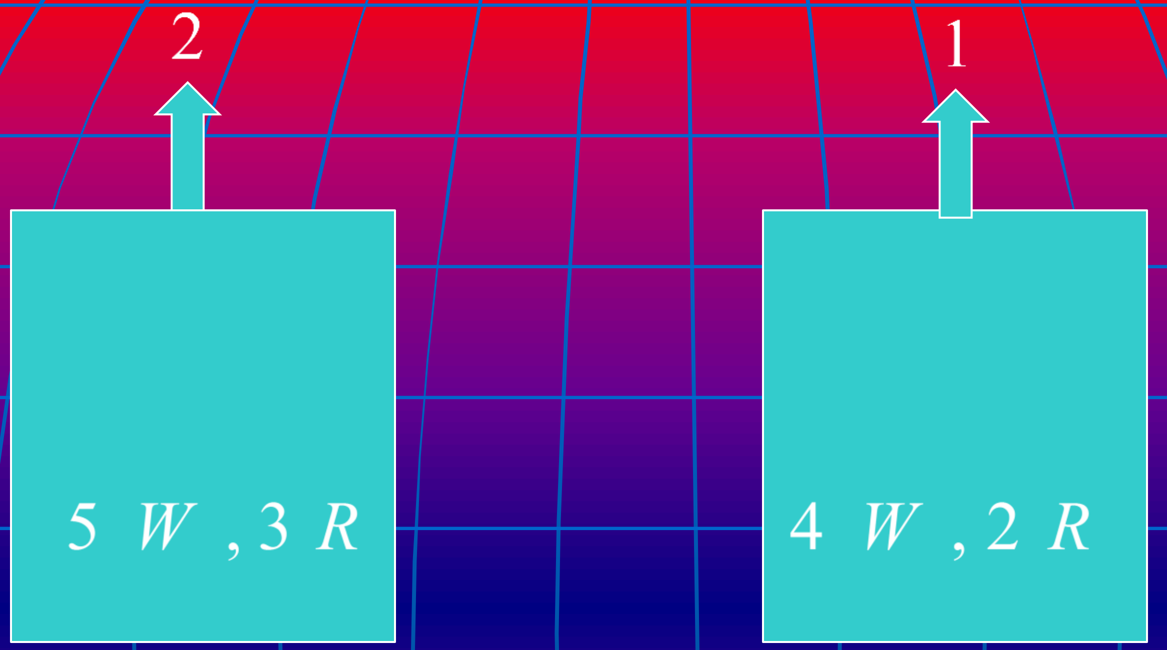
$$= {}^{2}C_{0} \times {}^{4}C_{1} \times {}^{4}C_{0} \times {}^{2}C_{1}$$

P (both are red) =

The number of favorable outcomes for getting different colour

$$= {}^2C_1 \times {}^4C_0 \times {}^4C_0 \times {}^2C_1 + {}^2C_0 \times {}^4C_1 \times {}^4C_1 \times {}^2C_0$$

P (different colour) =



4 .The number of possible outcomes

$$= {}^{8}C_{2} \times {}^{6}C_{1}$$

The number of possible outcomes $= {}^{8}C_{2} \times {}^{6}C_{1}$

The number of ways that exactly two white

$$= {}^{5}C_{2} \times {}^{3}C_{0} \times {}^{4}C_{0} \times {}^{2}C_{1} + {}^{5}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{1} \times {}^{2}C_{0}$$

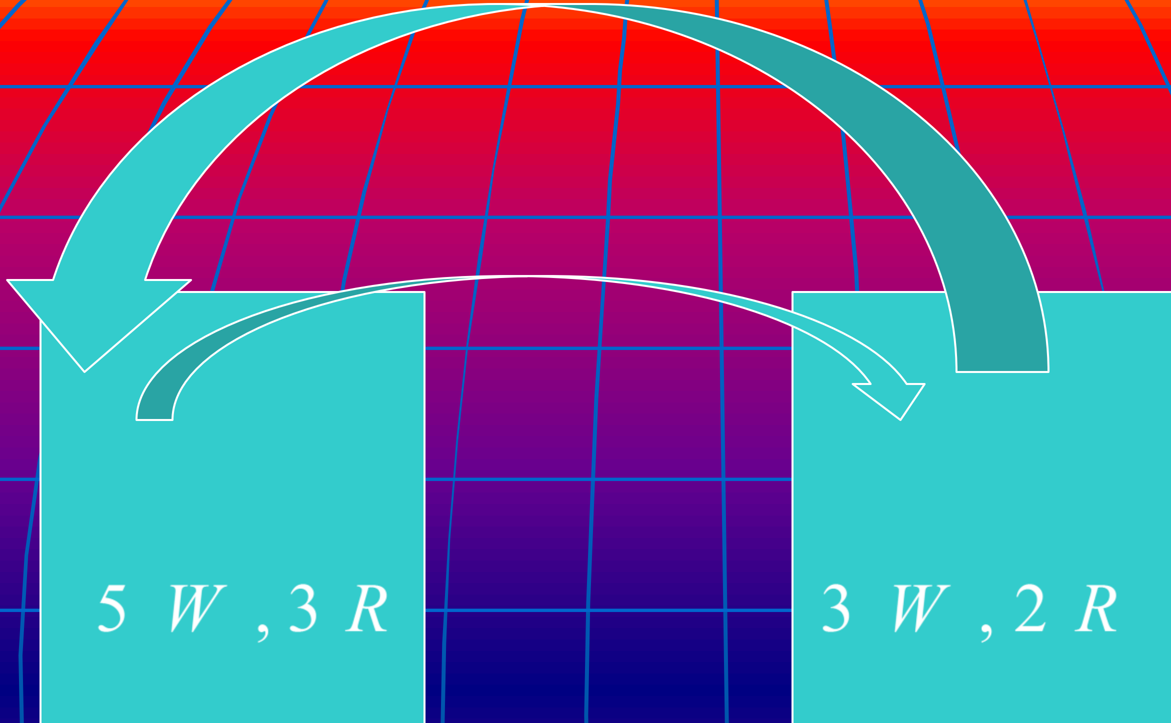
P (exactly two white) $= \dfrac{\qquad}{{}^{8}C_{2} \times {}^{6}C_{1}}$

The number of ways that at least two white

$$= {}^{5}C_{2} \times {}^{3}C_{0} \times {}^{4}C_{0} \times {}^{2}C_{1} + {}^{5}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{1} \times {}^{2}C_{0}$$

$$+ {}^{5}C_{2} \times {}^{3}C_{0} \times {}^{4}C_{1} \times {}^{2}C_{0}$$

P (at least two white) $= \dfrac{\qquad}{{}^{8}C_{2} \times {}^{6}C_{1}}$



7 .The number of possible outcomes

$$= {}^{8}C_{1} \times {}^{6}C_{1}$$

7 .The number of possible outcomes

$$= {}^{8}C_{1} \times {}^{6}C_{1}$$

The number of favorable outcomes such that A still contain 5 white and 3 red balls

$$= {}^{5}C_{1} \times {}^{3}C_{0} \times {}^{4}C_{1} \times {}^{2}C_{0} + {}^{5}C_{0} \times {}^{3}C_{1} \times {}^{3}C_{0} \times {}^{3}C_{1}$$

P (A still contain 5 white and 3 red) =

The number of favorable outcomes that taken red from B

$$= {}^5C_1 \times {}^3C_0 \times {}^4C_0 \times {}^2C_1 + {}^5C_0 \times {}^3C_1 \times {}^3C_0 \times {}^3C_1$$

P (taken red from B) =

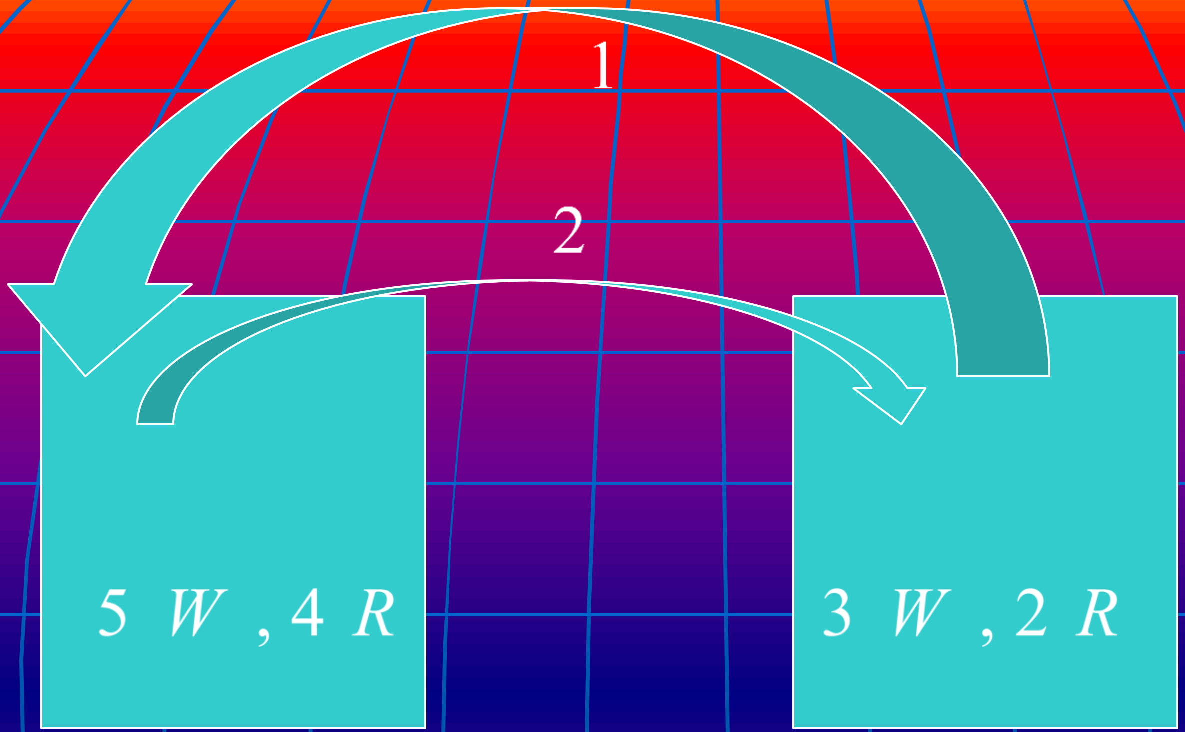
The number of favorable outcomes such that B still contain 4 white and 1 red balls $= {}^5C_1 \times {}^3C_0 \times {}^4C_0 \times {}^2C_1$

P (B still contain 4 white and 1 red) =

The number of favorable outcomes such that A still contain 6 white

$$= {}^5C_0 \times {}^3C_1 \times {}^3C_1 \times {}^3C_0$$

P (A still contain 6 white) =



.The number of possible outcomes

$$= {}^{9}C_{2} \times {}^{7}C_{1}$$

The number of possible outcomes $= {}^{9}C_{2} \times {}^{7}C_{1}$

The number of ways that A still 5 white

$$= {}^{5}C_{0} \times {}^{4}C_{2} \times {}^{3}C_{0} \times {}^{4}C_{1} + {}^{5}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{3}C_{0}$$

P (A still 5 white) $= \dfrac{\quad}{{}^{9}C_{2} \times {}^{7}C_{1}}$

The number of ways that A still 4 white

$$= {}^{5}C_{2} \times {}^{4}C_{0} \times {}^{5}C_{1} \times {}^{2}C_{0} + {}^{5}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{0} \times {}^{3}C_{1}$$

9 . The number of possible blood groups

$$= {}^{3}C_{3} + {}^{3}C_{2} + {}^{3}C_{1} + {}^{3}C_{0}$$

$$(AB^{+})\ (AB, A^{+}, B^{+})\ (A^{-}, B^{-}, O^{+})\ (O^{-})$$

$$= 1 + 3 + 3 + 1 = 8$$

The probability that 3 of 5 blood donor will be A^{+}

$$= {}^{5}C_{3} \times \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{7}{8} \times \frac{7}{8}$$

10. The number of possible outcomes $= {}^{100}C_6$

The number of ways that two of them are defective

$$= {}^{90}C_4 \times {}^{10}C_2$$

P (exactly two white) $= \dfrac{{}^{90}C_4 \times {}^{10}C_2}{{}^{100}C_6}$

The number of ways that at least one defective

$$= {}^{90}C_5 \times {}^{10}C_1 + {}^{90}C_4 \times {}^{10}C_2 + {}^{90}C_3 \times {}^{2}C_0$$

$$+ {}^{90}C_2 \times {}^{10}C_4 + {}^{90}C_1 \times {}^{10}C_5 + {}^{90}C_0 \times {}^{10}C_6$$

P (at least one defective) $= \dfrac{\qquad}{{}^{100}C_6}$

11. The number of possible outcomes $= 365^5$

The number of ways that different birthday

$$= {}^{365}P_5 = 365 \times 364 \times 363 \times 362 \times 361$$

P (different birthday) $= \dfrac{{}^{365}P_6}{365^5}$

P (at least tw of them have same birthday)

$$= 1 - \frac{{}^{365}P_5}{365^5}$$

12. The number of possible outcomes $= 2^{10}$

The number of ways that correct answer

$$= {}^{10}C_6$$

P (exactly 6 question correctly) $= \dfrac{{}^{10}C_6}{2^{10}}$

13. The number of possible outcomes $= {}^{20}C_{10}$

The number of ways that at least 8 words known

$$= {}^{12}C_8 \times {}^{8}C_2 + {}^{12}C_9 \times {}^{8}C_1 + {}^{12}C_{10} \times {}^{8}C_0$$

P (at least 8 words known) $= \dfrac{\qquad}{{}^{20}C_{10}}$

14. The number of possible outcomes $= {}^{5}C_{3}$

The number of ways that selected brand B

$$= {}^{4}C_{2}$$

P (selected brand B) $= \dfrac{{}^{4}C_{2}}{{}^{5}C_{3}}$

The number of ways that selected brand B and C

$$= {}^{3}C_{1}$$

P (selected brand B and C) $= \dfrac{{}^{3}C_{1}}{{}^{5}C_{3}}$

The number of ways that selected at least one of the two brands B and C

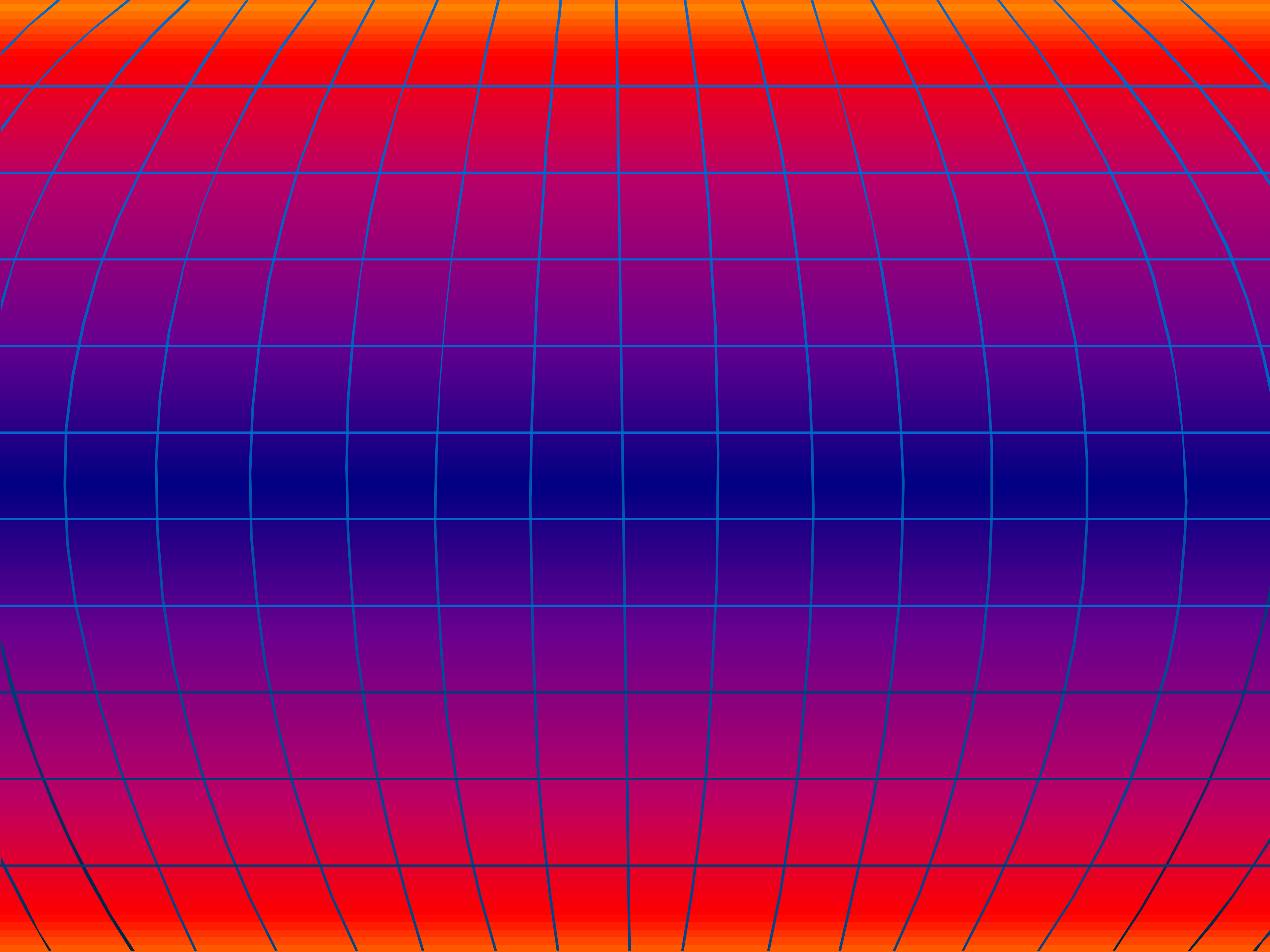
$$= {}^{3}C_{2} + {}^{3}C_{2} + {}^{3}C_{1}$$

$$or \quad = {}^{4}C_{2} + {}^{4}C_{2} - {}^{3}C_{1}$$

$$or \quad = {}^{2}C_{2} \times {}^{3}C_{1} + {}^{2}C_{1}\ {}^{3}C_{2}$$

P (at least one of the brand B and C)

$$= \frac{}{{}^{5}C_{3}}$$



Conditional Probability

Consider the following problem :

One hundred items consists of 20 defective and 80 non defective items. Suppose we choose two items

(a) with replacement

(b) without replacement

A = { the first item is defective }

B = { the second item is defective }

(i) If we are chosen with replacement

$$p(A)=\frac{20}{100}, P(A')=\frac{80}{100} \quad p(B)=\frac{20}{100}, P(B')=\frac{80}{100}$$

(ii) If we are chosen without replacement

$$p(A)=\frac{20}{100}, P(A')=\frac{80}{100} \qquad p(B)=?, P(B')=?$$

Let A and B be two events associated with an experiment. We denote by p (B/ A) the conditional probability of the event B given that A occurred.

$$p(B/A)=\frac{19}{99}$$

$$p(B/A')=\frac{20}{99}$$

$$p(B'/A)=\frac{80}{99}$$

$$p(B'/A')=\frac{79}{99}$$

Whenever we compute p(B / A) we are computing p (B) with respect to the reduce sample space A, rather than with respect to the original sample space

Example .

Two fair dice are tossed, the outcome being recoded as (x_1 , x_2) , where x_i is the outcome of i^{th} die $i = 1 , 2$. Hence the sample space S may be represented by the following of 36 equally likely outcomes.

$S = \{(1 , 1) , ..., (6 , 6) \}$

Consider the following two events

$A = \{ (x_1 , x_2) / x_1 + x_2 = 10 \}$

$B = \{ (x_1 , x_2) / x_1 > x_2 \}$

$A = \{(4,6),(5,5),(6,4)\}$

$B = \{(2,1),(3,1),(3,2),(4,1)$

$,(4,2),(4,3),(5,1),(5,2),(5,3)$

$,(5,4),(6,1),(6,2),(6,3),(6,4)$

$,(6,5)\}$

$$A \cap B = \{(6,4)\}$$

$$p(A) = \frac{3}{36}, \quad p(B) = \frac{15}{36}, \quad p(A \cap B) = \frac{1}{36}$$

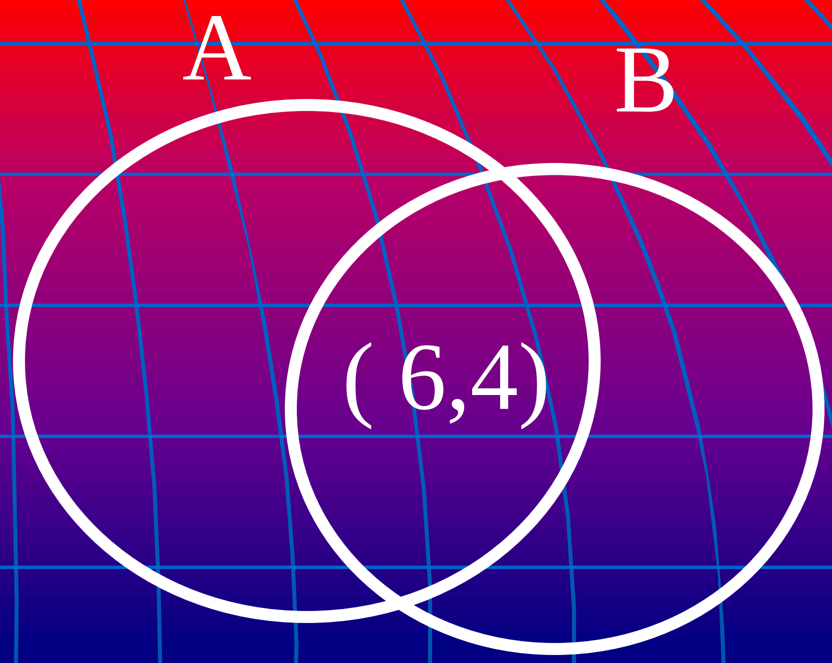
What about p (A / B)= ?

$$P(A/B)=\frac{1}{15}$$

$$p(A/B)=\frac{n(A\cap B)}{n(B)}$$

$$p(A/B)=\frac{n(A\cap B)/n(S)}{n(B)/n(S)}=\frac{p(A\cap B)}{P(B)}$$

$$//ly \qquad p(B/A)=\frac{p(A\cap B)}{p(A)}$$



Multiplication Theorem for Conditional Probability

$$*\,p(A/B)=\frac{P(A\cap B)}{P(B)}$$

$$p(A\cap B)=P(B)P(A/B)$$

$$*\,p(B/A)=\frac{P(A\cap B)}{P(A)}$$

$$p(A\cap B)=P(A)P(B/A)$$

$$p(A\cap B\cap C)=P(A)P(B/A)P(C/A\cap B)$$

A class has 12 boys and 4 girls. If three students are selected at random from the class what is the probability that they are all boys

Let A = the first student is boy

B = the second student is boy

C = the third student is boy

$$p(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$$

$$= \frac{12}{16} \times \frac{11}{15} \times \frac{10}{14}$$

In a certain school, 25% of the students failed mathematics, 15 % of the student failed chemistry, and 10% of the students failed both mathematics and chemistry. A student is selected at random

(i) If he failed chemistry, what is the probability that he failed mathematics ?

(ii) If he failed mathematics, what is the probability that he failed chemistry?

(iii) What is the probability that he failed mathematics or chemistry?

Let M = {students who failed mathematics}

C = { students who failed chemistry }

P (M) = 0.25 , P (C) = 0.15 ,

$$p(M \cap C) = 0.10$$

(i)The probability that a student failed mathematics, given that he has failed chemistry is

$$p(M/C) = \frac{P(M \cap C)}{P(C)} = \frac{0.10}{0.15} = \frac{2}{3}$$

(ii)The probability that a student failed chemistry, given that he has failed mathematics is

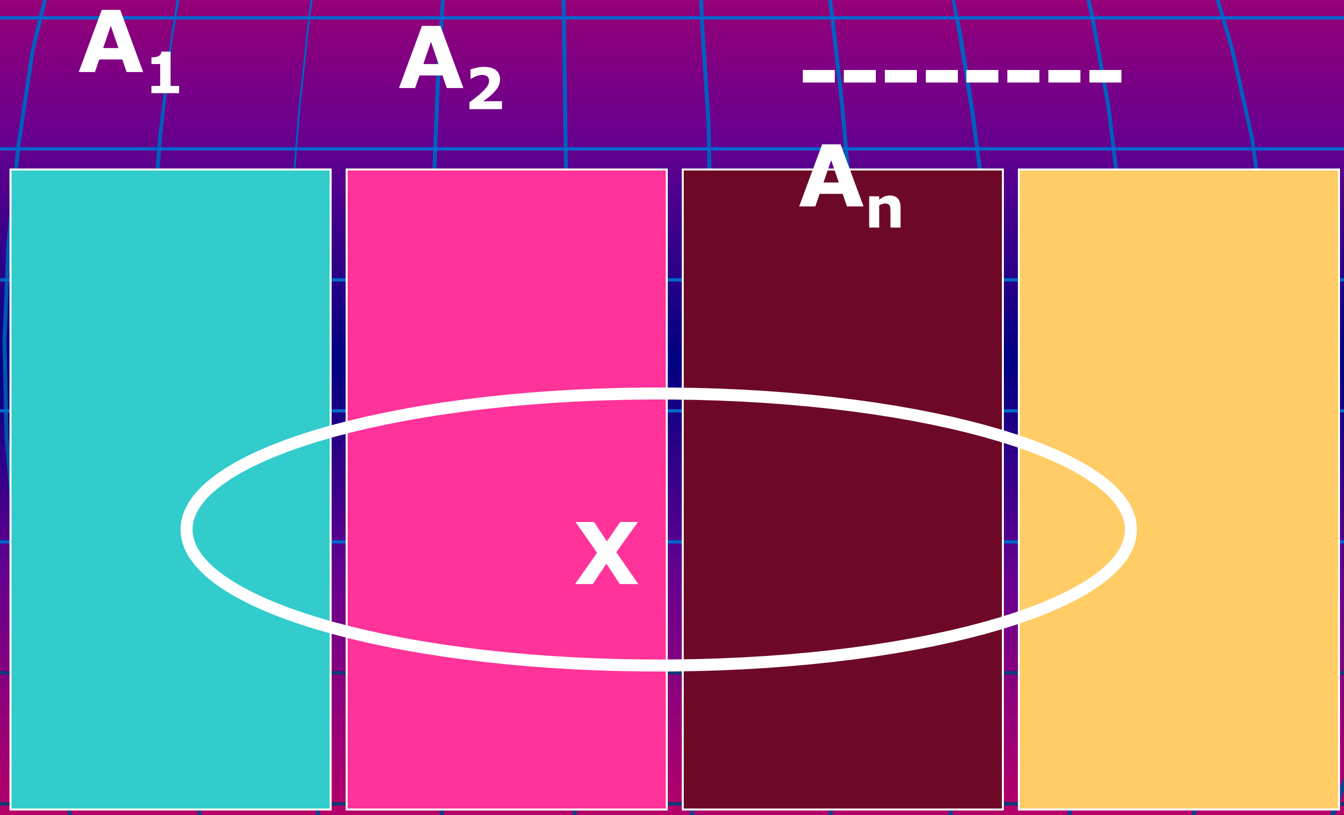
$$p(C/M) = \frac{P(C \cap M)}{P(M)} = \frac{0.10}{0.25} = \frac{2}{5}$$

(iii) P(M U C)

$$= P(M) + P(C) - P(M \cap C)$$

$$= 0.25 + 0.15 - 0.10 = 0.3 = \frac{3}{10}$$

Let X be an event with respect to S and $A_1, A_2, \ldots\ldots, A_n$ be a partition of S. Then we may write



$$X = (A_1 \cap X) \cup (A_2 \cap X) \cup \ldots\ldots \cup (A_n \cap X)$$

Let X be an event with respect to S and $A_1, A_2, \ldots\ldots, A_n$ be a partition of S. Then we may write

$$X = (A_1 \cap X) \cup (A_2 \cap X) \cup \ldots\ldots \cup (A_n \cap X)$$

$$P(X) = P(A_1 \cap X) + P(A_2 \cap X) + \ldots\ldots\ldots + P(A_n \cap X)$$

$$\mathrm{P(X) = P(A_1)P(X/A_1) + \ldots\ldots + P(A_n)P(X/A_n)}$$

$$P(X) = \sum_{i=1}^{n} P(A_i)P(X|A_i)$$

Baye's Theorem

$$P(X) = P(A_1 \cap X) + P(A_2 \cap X) + \ldots + P(A_n \cap X)$$

$$P(X) = \sum_{i=1}^{n} P(A_i) P(X / A_i)$$

$$P(A_i / X) = \frac{P(A_i \cap X)}{P(X)}$$

$$P(A_i / X) = \frac{P(A_i) P(X / A_i)}{P(X)}$$

$$(I) \qquad p(B/A)=\frac{p(A\cap B)}{p(A)}$$

$$p(A/B)=\frac{p(A\cap B)}{p(B)}$$

$$p(X/Z)=\frac{p(X\cap Z)}{p(Z)}$$

$$p(A/X)=\frac{p(A\cap X)}{p(X)}$$

$(II)\quad A \Rightarrow B$

$$p(B \cap A) = P(A)P(B/A)$$

$$B \Rightarrow A$$

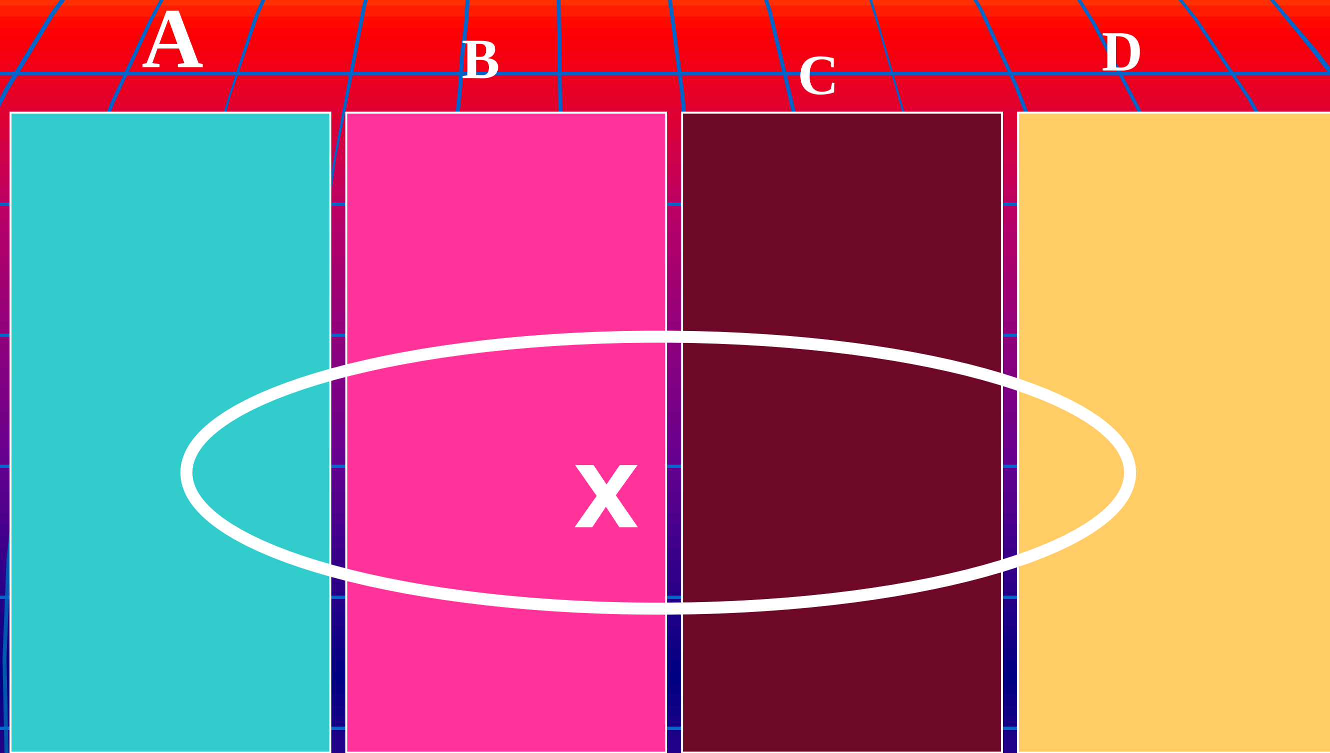
$$p(A \cap B) = P(B)P(A/B)$$

$$Z \Rightarrow X$$

$$p(X \cap Z) = P(Z)P(X/Z)$$

$$C \Rightarrow X$$

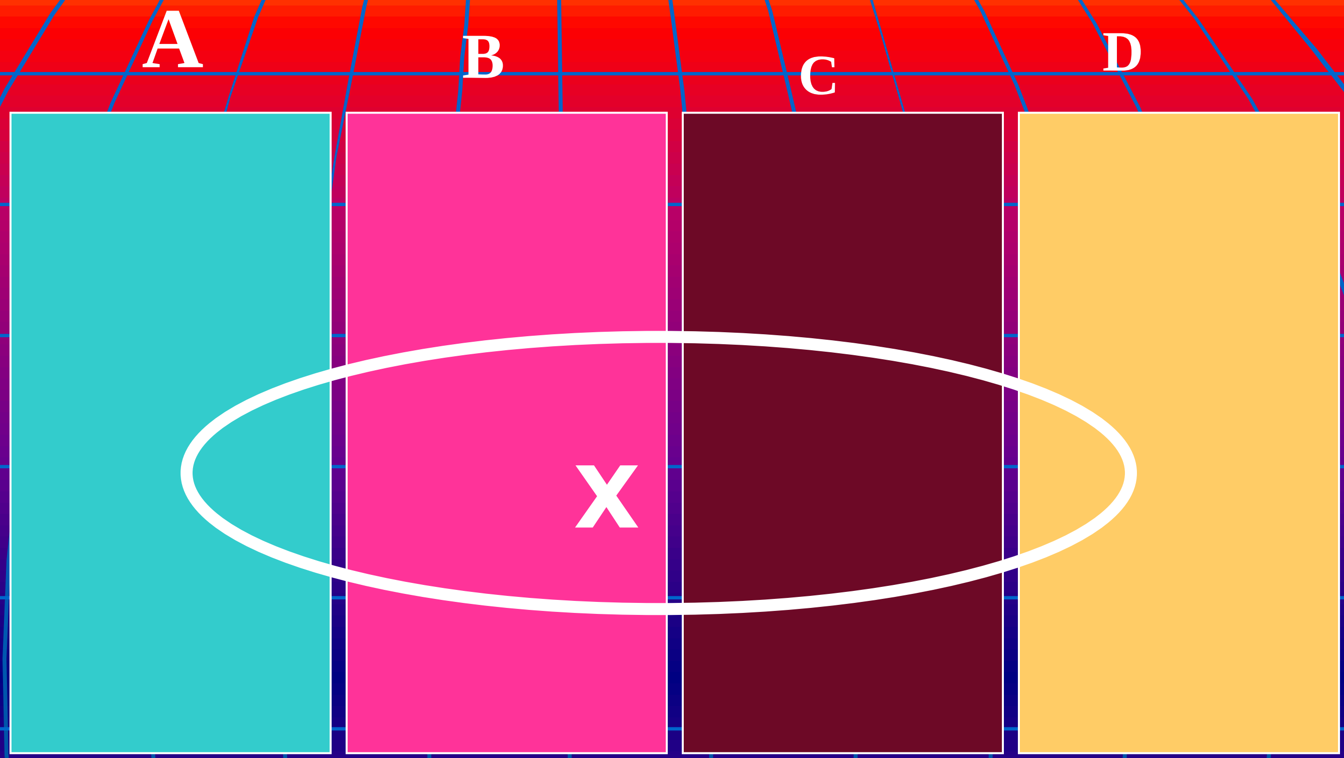
$$p(C \cap X) = P(C)P(X/C)$$



$$X = (A \cap X) \cup (B \cap X) \cup (C \cap X) \cup (D \cap X)$$

$$P(X) = P(A \cap X) + P(B \cap X) + P(C \cap X) + P(D \cap X)$$

$$P(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C) + P(D)P(D/X)$$



$$P(C/X) = \frac{P(C \cap X)}{P(X)}$$

$$P(C/X) = \frac{P(C)P(X/C)}{P(X)}$$

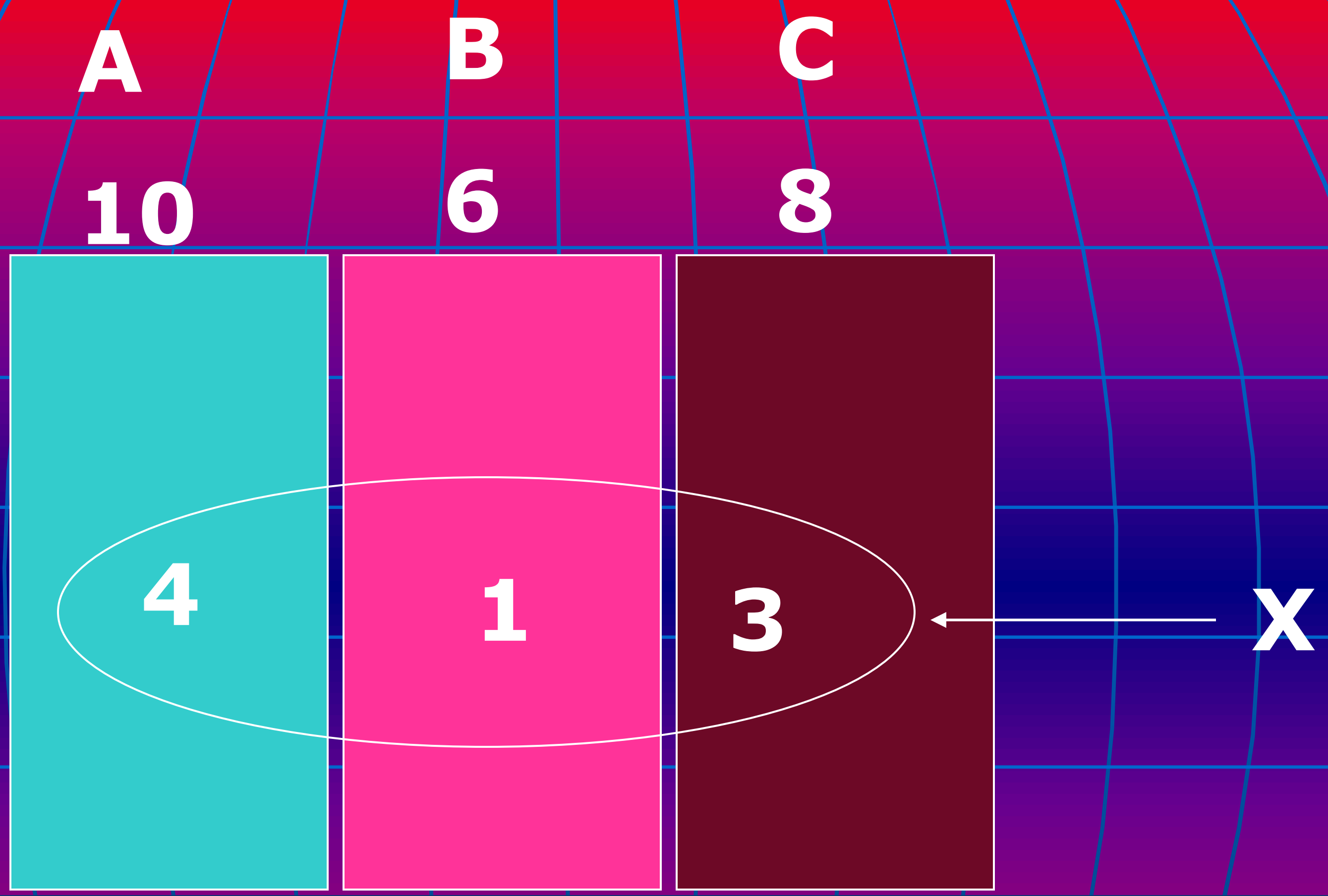
We are given three boxes as follows:

Box A has 10 light bulbs of which 4 are defective.

Box B has 6 light bulbs of which 1 is defective

Box C has 8 light bulbs of which 3 are defective

We selected a box at random and draw a bulb at random. What is the probability that the bulb is defective.


A
B
C
10
6
8
4
1
3
X

$$p(A)=\frac{1}{3}\ ,\ P(B)=\frac{1}{3}\ ,\ P(C)=\frac{1}{3}$$

Let X be the event that draw a defective

$$P(X/A)=\frac{2}{5}\ ,\ P(X/B)=\frac{1}{6}\ ,\ P(X/C)=\frac{3}{8}$$

$$P(X'/A)=\frac{3}{5}\ ,\ P(X'/B)=\frac{5}{6}\ ,\ P(X'/C)=\frac{5}{8}$$

$$X=(A\cap X)\cup(B\cap X)\cup(C\cap X)$$

$$P(X)=P(A\cap X)+P(B\cap X)+P(C\cap X)$$

$$P(X)=P(A\cap X)+P(B\cap X)+P(C\cap X)$$

$$P(X)=P(A).P(X/A)+P(B).P(X/B)$$
$$+P(C)P(X/C)$$

$$=\frac{1}{3}\times\frac{2}{5}+\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{3}{8}$$

$$=\frac{2}{15}+\frac{1}{18}+\frac{1}{8}$$

$$=\frac{113}{360}$$

Find the probability that the bulb is non defective

$$X' = (A \cap X') \cup (B \cap X') \cup (C \cap X')$$

or

$$P(X') = 1 - P(X)$$

$$= 1 - \frac{113}{360}$$

$$= \frac{247}{360}$$

Suppose a bulb is draw at random and is found to be defective. Find the probability that the bulb was draw by box A.

$$P(A/X) = \frac{P(A \cap X)}{P(X)} = \frac{P(A)P(X/A)}{P(X)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{113}{360}} = \frac{48}{113}$$

Suppose a bulb is draw at random and is found to be non defective. Find the probability that the bulb was draw by box A.

$$P(A/X')=\frac{P(A\cap X')}{P(X')}=\frac{P(A)P(X'/A)}{P(X')}$$

$$=\frac{\frac{1}{3}\times\frac{3}{5}}{\frac{247}{360}}\qquad=\frac{72}{247}$$

Suppose a bulb is draw at random and is found to be defective. Find the probability that the bulb was not drawn by box A.

$$P(A'/X) = 1 - P(A/X)$$

Suppose a bulb is draw at random and is found to be non defective. Find the probability that the bulb was not drawn by box A.

$$P(A'/X') = 1 - P(A/X')$$

A coin weighted so that P (H) = 2/3 and P (T) = 1/3, is tossed. If head appears, then a number is selected at random from the number 1 through 9; if tail appears, then a number is selected at random from the number 1 through 5. Find the probability that an even number is selected.

$$P(H)=\frac{2}{3} \qquad P(T)=\frac{1}{3}$$

Let X be the event that an even number is selected

$$P(X/H)=\frac{4}{9} \qquad P(X/T)=\frac{2}{5}$$

$$P(H)=\frac{2}{3} \qquad P(T)=\frac{1}{3}$$

Let X be the event that an even number is selected

$$P(X/H)=\frac{4}{9} \qquad P(X/T)=\frac{2}{5}$$

$$X = (H \cap X) \cup (T \cap X)$$

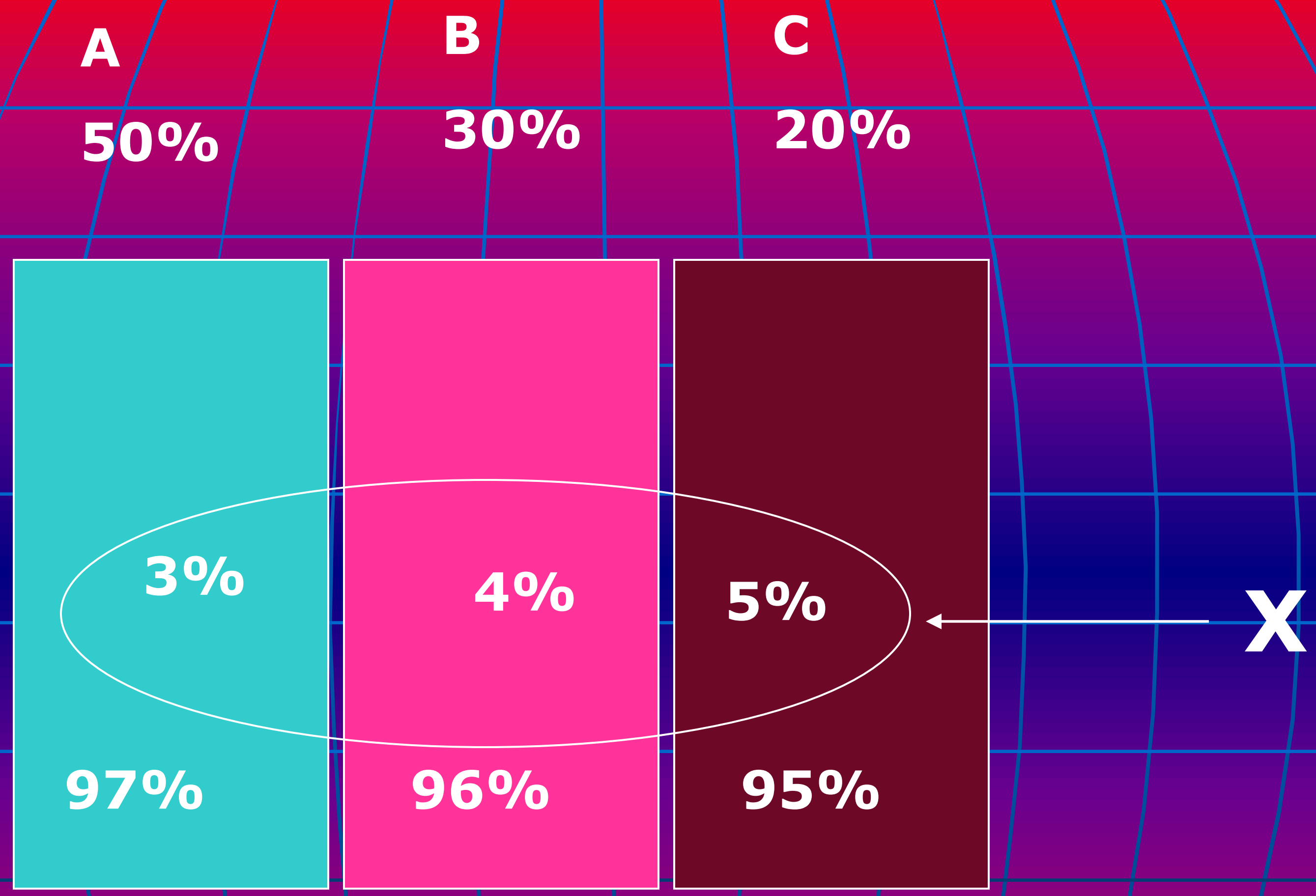
$$P(X)=P(H \cap X)+P(T \cap X)$$

$$P(X)=P(H).P(X/H)+P(T).P(X/T)$$

If the number is even, what is the probability that the coin was head?

$$P(H/X)=\frac{P(H\cap X)}{P(X)}=\frac{P(H)P(X/H)}{P(X)}$$

Three machines A , B and C produce respectively 50 % , 30 % and 20 % of the total number of items of a factory. The percentages of defective output of these machines are 3 % , 4 % and 5 % . If an items in selected at random, find the probability that the item is defective. Suppose that an item is selected at random and found to be defective.


A
50%
B
30%
C
20%
3%
4%
5%
X
97%
96%
95%

$$P(A)=\frac{50}{100} \qquad P(B)=\frac{30}{100} \qquad P(C)=\frac{20}{100}$$

Let X be the event that the item is defective

$$P(X|A)=\frac{3}{100} \qquad P(X|B)=\frac{4}{100} \qquad P(X|C)=\frac{5}{100}$$

$$X=(A\cap X)\cup(B\cap X)\cup(C\cap X)$$

$$P(X)=P(A\cap X)+P(B\cap X)+P(C\cap X)$$

$$P(X)=P(A).P(X|A)+P(B).P(X|B)$$

$$+P(C).P(X|C)$$

$$\mathrm{P(X)}=\frac{5}{10}\times\frac{3}{100}+\frac{3}{10}\times\frac{4}{100}+\frac{2}{10}\times\frac{5}{100}$$

$$P(A/X) = \frac{P(A \cap X)}{P(X)}$$

$$P(A/X) = \frac{P(A)P(X/A)}{P(X)}$$

$$P(A)=\frac{50}{100} \qquad P(B)=\frac{30}{100} \qquad P(C)=\frac{20}{100}$$

Let X be the event that the item is defective

$$P(X'|A)=\frac{97}{100} \qquad P(X'|B)=\frac{96}{100} \qquad P(X'|C)=\frac{95}{100}$$

$$X'=(A\cap X')\cup(B\cap X')\cup(C\cap X')$$

$$P(X')=P(A\cap X')+P(B\cap X')+P(C\cap X')$$

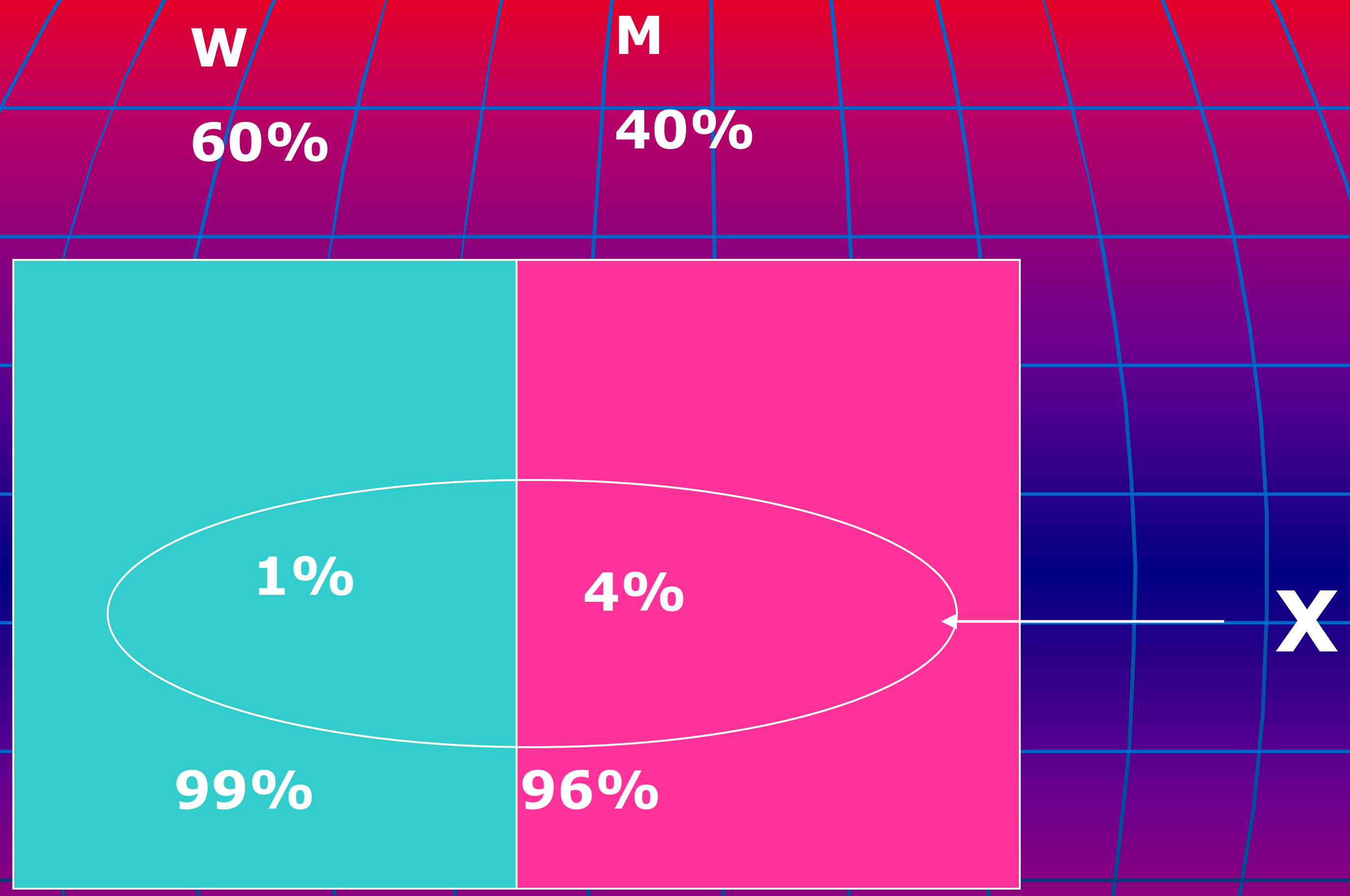
$$P(X')=P(A).P(X'/A)+P(B).P(X'/B)+P(C).P(X'/C)$$

(or) $P(X')=1-P(X)$

$$P(A/X') = \frac{P(A \cap X')}{P(X')}$$

$$P(A/X') = \frac{P(A)P(X'/A)}{P(X')}$$

In a school4% of the men and 1 % of the women are taller than 6 ft. Further more, 60 % of the students are women. Now if a student is selected at random and is taller than 6 ft, what is the probability that the student is a women.


W
60%
M
40%
1%
4%
X
99%
96%

$$P(M)=\frac{40}{100}$$ $$P(W)=\frac{60}{100}$$

Let X be the event that the student who taller than 6ft.

$$P(X|M)=\frac{4}{100}$$ $$P(X|W)=\frac{1}{100}$$

$$X=(M\cap X)\cup(W\cap X)$$

$$P(X)=P(M\cap X)+P(W\cap X)$$

$$P(X)=P(M).P(X/M)+P(W).P(X/W)$$

$$P(W/X) = \frac{P(W \cap X)}{P(X)}$$

$$P(W/X) = \frac{P(W)P(X/W)}{P(X)}$$

We are given three boxes as follows:
Box A has 12 white 4 red and 4 blue balls .
Box B has 6 white 2 red and 2 blue balls.
Box C has 9 white 4 red and 2 blue balls.
We selected a box at random and draw a ball at random. Suppose a ball is draw at random and is found to be white. Find the probability that the ball was not draw by box A.

$$p(A)=\frac{1}{3}\ ,\ P(B)=\frac{1}{3}\ ,\ P(C)=\frac{1}{3}$$

Let W be the event that draw a white ball

$$P(W/A)=\frac{12}{20}\ ,\ P(W/B)=\frac{6}{10}\ ,\ P(W/C)=\frac{9}{15}$$

$$P(W'/A)=\frac{8}{20}\ ,\ P(W'/B)=\frac{4}{10}\ ,\ P(W'/C)=\frac{6}{15}$$

$$W=(A\cap W)\cup(B\cap W)\cup(C\cap W)$$

$$P(W)=P(A\cap W)+P(B\cap W)+P(C\cap W)$$

$$P(W) = P(A \cap W) + P(B \cap W) + P(C \cap W)$$

$$P(W) = P(A).P(W/A) + P(B).P(W/B) + P(C)P(W/C)$$

$$= \frac{1}{3} \times \frac{12}{20} + \frac{1}{3} \times \frac{6}{10} + \frac{1}{3} \times \frac{9}{15}$$

$$= \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

$$= \frac{3}{5}$$

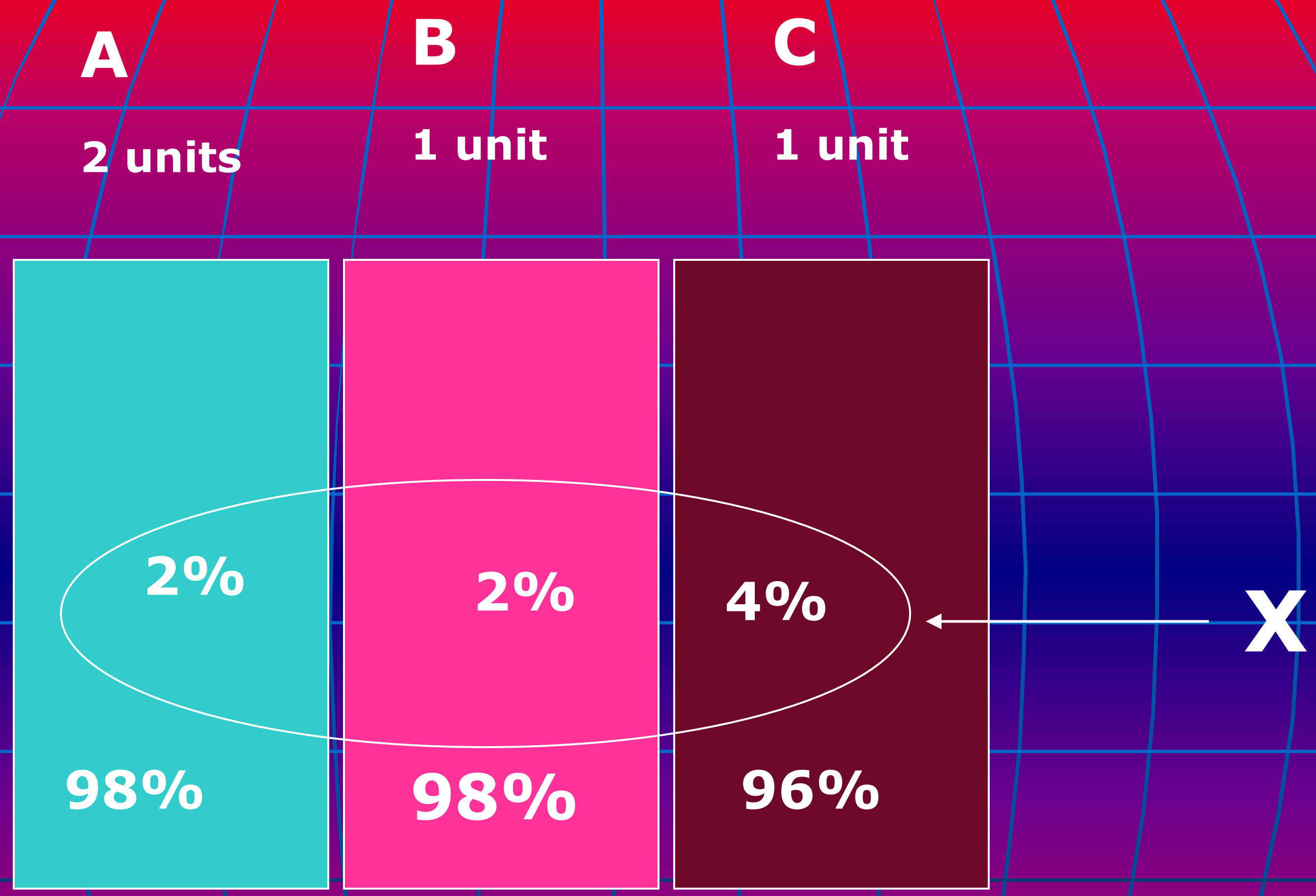
$$P(W')=1-P(W) \quad =1-\frac{3}{5} \quad =\frac{2}{5}$$

$$P(A/W)=\frac{P(A\cap W)}{P(W)}=\frac{P(A)P(W/A)}{P(W)}$$

$$=\frac{\frac{1}{3}\times\frac{12}{20}}{\frac{3}{5}}$$

$$P(A'/W)=1-P(A/W)$$

A certain item is manufactories by factories, say A , B and C. It is known that A turns out twice as may item as B, and that B and C turns out the same number of items (during a specified production period) . It is also known that 2 % of items produced by A and B are defective, with 4 % of those manufactory by C are defective. All the times produced are put into one stock , and then one item is chosen at random. Suppose that an item is selected at random is found to be non defective. Find the probability that the item produced by A.


A
2 units
B
1 unit
C
1 unit
2%
2%
4%
X
98%
98%
96%

$$P(A)=\frac{1}{2} \qquad P(B)=\frac{1}{4} \qquad P(C)=\frac{1}{4}$$

Let X be the event that the item is defective

$$P(X|A)=\frac{2}{100} \qquad P(X|B)=\frac{2}{100} \qquad P(X|C)=\frac{4}{100}$$

$$P(X'|A)=\frac{98}{100} \qquad P(X'|B)=\frac{98}{100} \qquad P(X'|C)=\frac{96}{100}$$

$$X=(A\cap X)\cup(B\cap X)\cup(C\cap X)$$

$$P(X)=P(A\cap X)+P(B\cap X)+P(C\cap X)$$

$$P(X)=P(A).P(X|A)+P(B).P(X|B)$$

$$+P(C).P(X|C)$$

$$P(X)=\frac{1}{2}\times\frac{2}{100}+\frac{1}{4}\times\frac{2}{100}+\frac{1}{4}\times\frac{4}{100}$$

$$P(X)=\frac{1}{40} \qquad P(X')=1-\frac{1}{40}=\frac{39}{40}$$

$$P(A/X')=\frac{P(A\cap X')}{P(X')}$$

$$P(A/X')=\frac{P(A)P(X'/A)}{P(X')}$$

There are five urns, and they are numbered 1 to 5. Each urn contains 10 balls. Urn i has i defective balls, i = 1 , 2 , 3 , 4, 5. Consider the following random experiment. First an urn is selected at random and then a ball is selected at random from the selected urn.(i) What is the probability that a selected ball is defective.

(ii) If we have already selected ball and noted that is defective, what is the probability that is not come from urn 5.

$P(U_1)=\frac{1}{5}$ $P(U_2)=\frac{1}{5}$ $P(U_3)=\frac{1}{5}$ $P(U_4)=\frac{1}{5}$ $P(U_5)=\frac{1}{5}$

Let X be the event that the item is defective

$P(X|U_1)=\frac{1}{10}$ $P(X|U_2)=\frac{2}{10}$ $P(X|U_3)=\frac{3}{10}$

$P(X|U_4)=\frac{4}{10}$ $P(X|U_5)=\frac{5}{10}$

$$X=(U_1\cap X)\bigcup(U_2\bigcap X)\bigcup(U_3\bigcap X)(U_4\cap X)(U_5\cap X)$$

$$P(X)=P(U_1)P(X/U_1)+P(U_2)P(X/U_2)$$

$$+P(U_3).P(X|U_3)+P(U_4).P(X|U_4)$$

$$+P(U_5).P(X|U_5)$$

$$P(X)=P(U_1)P(X/U_1)+P(U_2)P(X/U_2)$$

$$+P(U_3).P(X/U_3)+P(U_4).P(X/U_4)$$

$$+P(U_5).P(X/U_5)$$

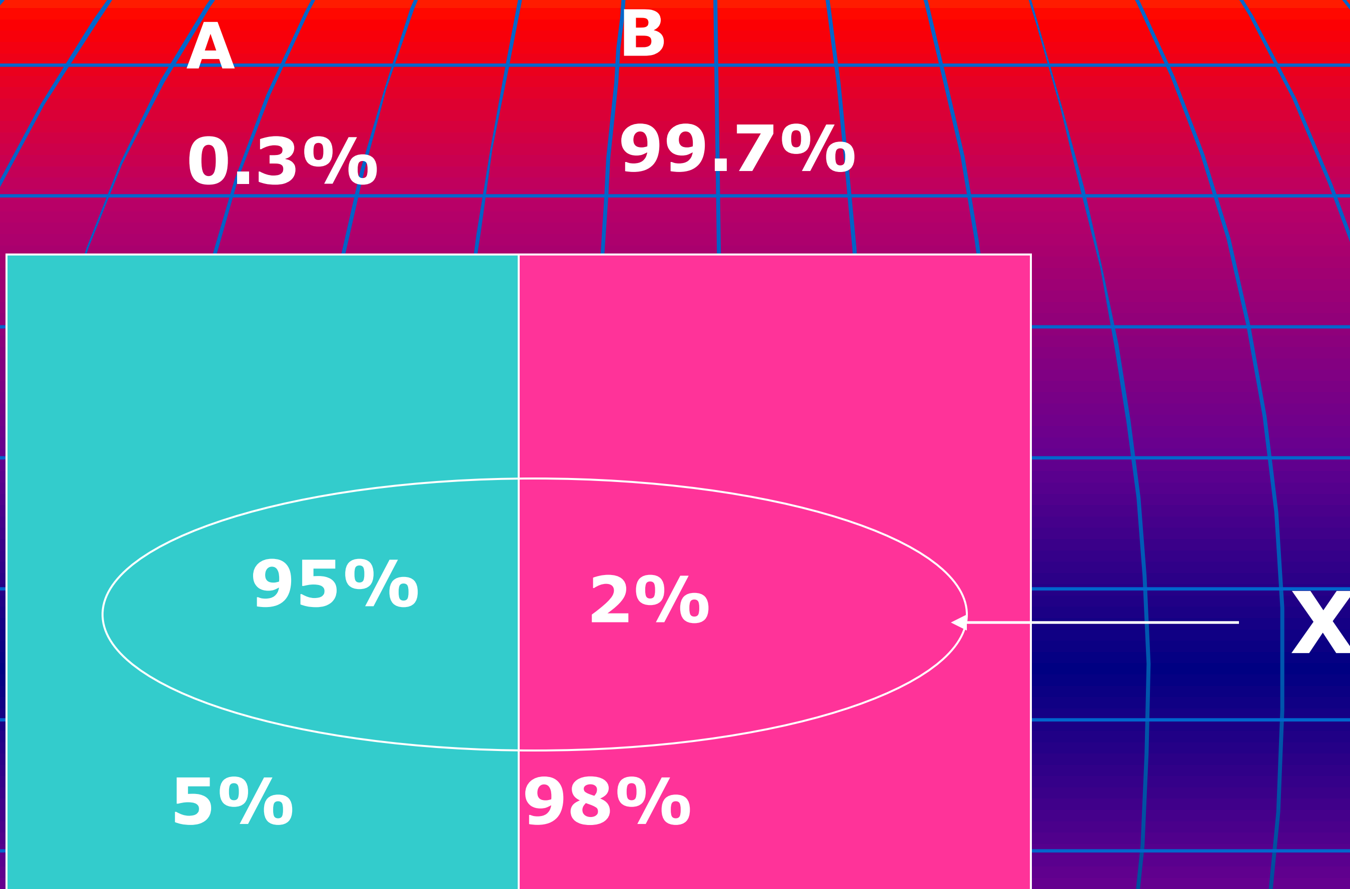
$$P(X)=\frac{1}{5}\times\frac{1}{10}+\frac{1}{5}\times\frac{2}{10}+\frac{1}{5}\times\frac{3}{10}+\frac{1}{5}\times\frac{4}{10}+\frac{1}{5}\times\frac{5}{10}$$

$$P(X)=\frac{3}{10}$$

$$P(U_5/X)=\frac{P(U_5\cap X)}{P(X)}=\frac{P(U_5)P(X/U_5)}{P(X)}$$

$$P(U_5'/X)=1-P(U_5/X)$$

If a certain desease is present, then a blood test will reveal it 95 % of the time. But the test will also indicate the present of the disease 2 % of the time when in fact the person tested is free of the disease; that is, the test gives a false positive 2 % of the time. If 0.3 % of the general population actually has the disease, what is the probability that a person chosen at random from the population has the disease given that he or she is tested positive.



Let A be the person with a certain disease

Let B be the person without a certain disease

$$P(A)=\frac{3}{1000} \qquad P(B)=\frac{997}{1000}$$

Let X be persons with certain disease by a blood test

$$P(X/A)=\frac{95}{100} \qquad P(X/B)=\frac{2}{100}$$

$$X=(A\cap X)\bigcup(B\bigcap X)$$

$$P(X)=P(A)P(X/A)+P(B)P(X/B)$$

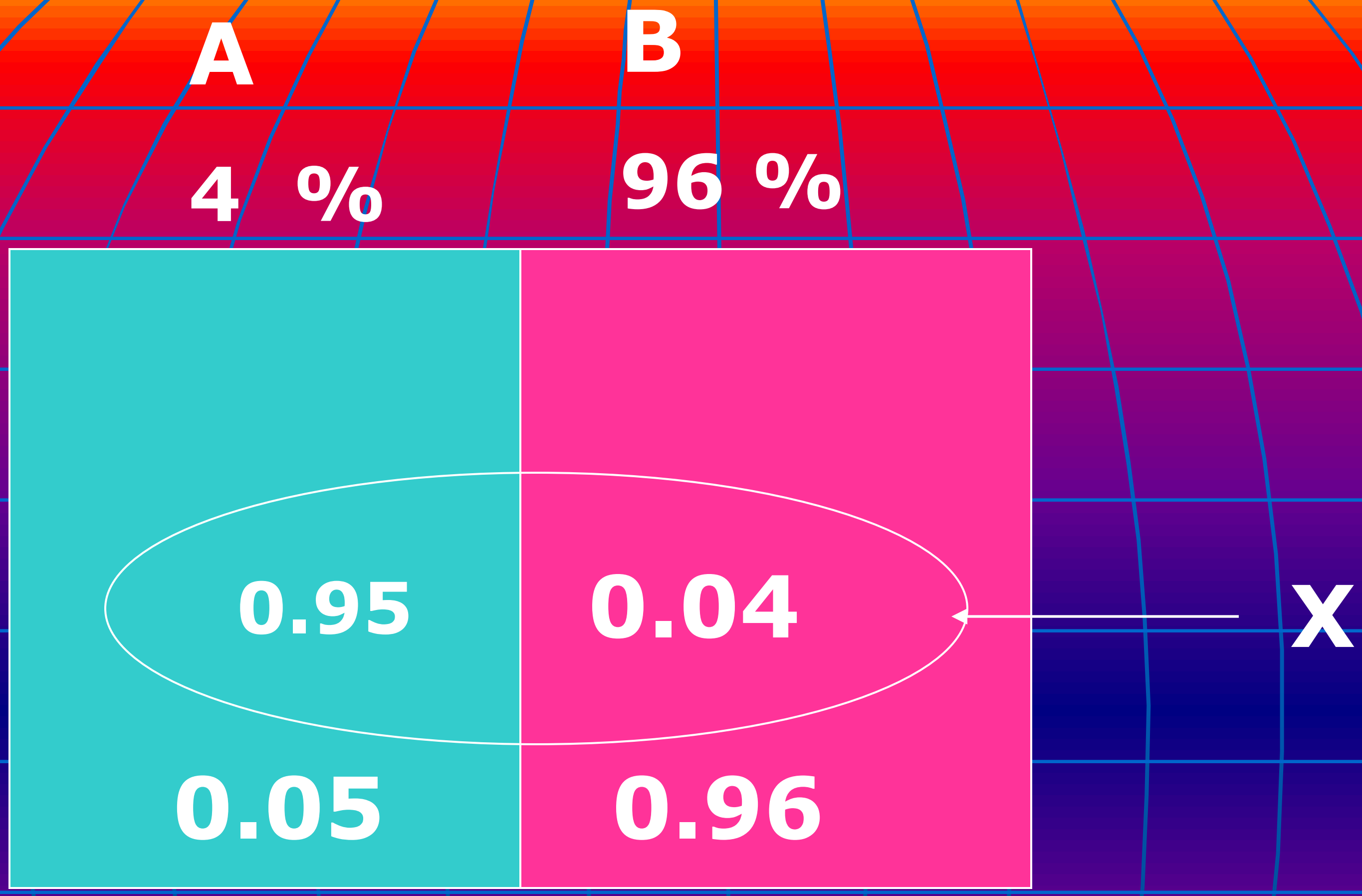
$$P(X) = P(A)P(X/A) + P(B)P(X/B)$$

$$P(X) = \frac{3}{1000} \times \frac{95}{100} + \frac{997}{1000} \times \frac{2}{100}$$

$$P(X) = \frac{285 + 1994}{100000}$$

$$P(A/X) = \frac{P(A \cap X)}{P(X)} = \frac{P(A)P(X/A)}{P(X)}$$

A medical test has been designed to detect the presence of certain disease. Among those who have the disease, the probability that the disease will be detected by the test is 0.95. However, the probability that the disease will erroneously indicate the present of the disease in those who do not actually have it is 0.04. It is estimated that 4 % of the population who take this test have the disease. If the test administerted to an induvidual is positive, what is the probability that the person actually has the disease ?



Let A be the person with disease

Let B be the prrson without disease

$$P(A)=\frac{4}{100} \qquad P(B)=\frac{96}{100}$$

Let X be persons with disease by the test

$$P(X/A)=\frac{95}{100} \qquad P(X/B)=\frac{4}{100}$$

$$P(X'/A)=\frac{5}{100} \qquad P(X'/B)=\frac{96}{100}$$

$$X=(A\cap X)\cup(B\cap X)$$

$$P(X)=P(A)P(X/A)+P(B)P(X/B)$$

$$P(X) = P(A)P(X/A) + P(B)P(X/B)$$

$$P(X) = \frac{4}{100} \times \frac{95}{100} + \frac{96}{100} \times \frac{4}{100}$$

$$P(X) = \frac{\quad}{10000}$$

$$P(A/X) = \frac{P(A \cap X)}{P(X)} = \frac{P(A)P(X/A)}{P(X)}$$

Based on data obtained from the institute of Dental Research. It has been determined that 42 % of 12-years olds have never had a cavity, 34 % of 13-year-olds have never had a cavity, and 28 % of 14-year-olds have never had a cavity. Suppose a child is selected at random from a group of 24 junior high school students that includes six 12-year-olds, eight 13-year-olds. and ten 14-year-olds. If this child does not have a cavity, what is the probability that the child is 14-year-old ?

	A	B	C
	12 years	13 years	14 years
	6	8	10
X	42%	34%	28%
	58%	66%	72%

Let A be the 12-year-olds child

Let B be the 13-year-olds child

Let C be the 14-year-olds child

Let X be the child who never had cavity

$$P(A)=\frac{6}{24}=\frac{1}{4} \qquad P(B)=\frac{8}{24}=\frac{1}{3} \qquad P(C)=\frac{10}{24}=\frac{5}{12}$$

$$P(X|A)=\frac{42}{100} \qquad P(X|B)=\frac{34}{100} \qquad P(X|C)=\frac{28}{100}$$

$$P(X'|A)=\frac{58}{100} \qquad P(X'|B)=\frac{66}{100} \qquad P(X'|C)=\frac{72}{100}$$

$$X=(A\cap X)\cup(B\cap X)\cup(C\cap X)$$

$$P(X)=P(A\cap X)+P(B\cap X)+P(C\cap X)$$

$$P(X)=P(A).P(X|A)+P(B).P(X|B)$$

$$+P(C).P(X|C)$$

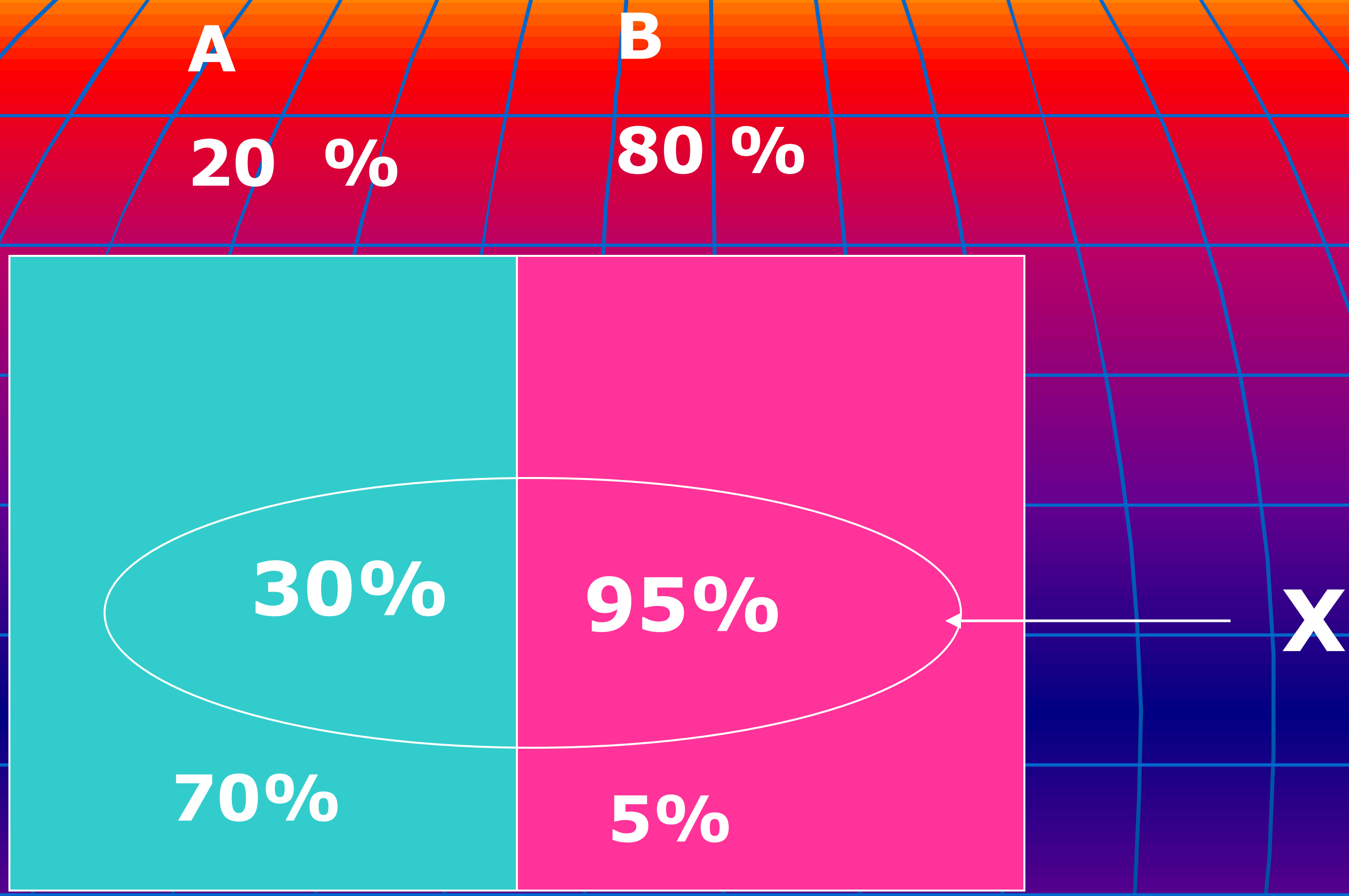
$$P(X)=\frac{1}{2}\times\frac{2}{100}+\frac{1}{4}\times\frac{2}{100}+\frac{1}{4}\times\frac{4}{100}$$

$$P(X)=\frac{1}{40}$$

$$P(C/X)=\frac{P(C\cap X)}{P(X)}$$

$$P(C/X)=\frac{P(C)P(X/C)}{P(X)}$$

A study was conducted among a certain group of union members whose health insurance policies second opinions prior to surgery. Of those member whose doctors advised them to have surgery, 20 % were informed by a second doctor that no surgery. Of these 70 % took the second doctor's opinion and did no go through with the surgery. Of the members who were advised to have surgery by both doctors, 95 % went through with the surgery. What is the probability that a union member who had surgery was adivised to do by a second doctor ?


A
20 %
B
80 %
30%
95%
X
70%
5%

Let A be the merber imform by a second doctor that no surgery was needed.

Let B be the merber imform by a second doctor that surgery was needed.

Let X be the merber who had surgery.

$$P(A)=\frac{20}{100}=\frac{2}{10}\quad P(B)=\frac{70}{100}=\frac{7}{10}$$

$$P(X/A)=\frac{30}{100}\qquad P(X/B)=\frac{95}{100}$$

$$P(X'/A)=\frac{70}{100}\qquad P(X'/B)=\frac{5}{100}$$

$$X=(A\cap X)\bigcup(B\bigcap X)$$

$$P(X)=P(A)P(X/A)+P(B)P(X/B)$$

$$P(B/X)=\frac{P(B\cap X)}{P(X)}=\frac{P(B)P(X/B)}{P(X)}$$

Raserchers weighed 1976 3-year-old from low-incomes famillyes in 20 cities. Each child is classified by race (white , black , Hispanic) and by weight (normal weight , over weight,or obese) The results are tabulated as follows:

		weight %		
Races	Childrem	Normal	over	Obese
White	400	68 %	18%	14 %
Black	1000	68 %	15 %	17 %
Hispanic	600	56 %	20 %	24 %

If a participant in the researcher is selected at random and is found to be obese, what is the probability that the 3-year-olds is white ? Hispanic ?

	W White 600	B Black 1000	H Hispanic 400
Normal Weight	68%	68%	56%
Overweight	18%	15%	20%
Obese	14%	17%	24%

Let W be the white childrem

Let B be the black childrem

Let H be the hispanic childrem

Let X be the obese childrem

$$P(W)=\frac{600}{2000}=\frac{3}{10}\quad P(B)=\frac{1000}{2000}=\frac{1}{2}\quad P(H)=\frac{400}{2000}=\frac{1}{5}$$

$$P(X|W)=\frac{14}{100}\quad P(X|B)=\frac{17}{100}\quad P(X|H)=\frac{24}{100}$$

$$P(X'|W)=\frac{86}{100}\quad P(X'|B)=\frac{83}{100}\quad P(X'|H)=\frac{76}{100}$$

$$X=(W\cap X)\cup(B\cap X)\cup(H\cap X)$$

$$P(X)=P(W\cap X)+P(B\cap X)+P(H\cap X)$$

$$P(X)=P(W).P(X|W)+P(B).P(X|B)$$
$$+P(H).P(X|H)$$