WELLCOME

TO

FOUNDATION YEAR (12 / 2019)

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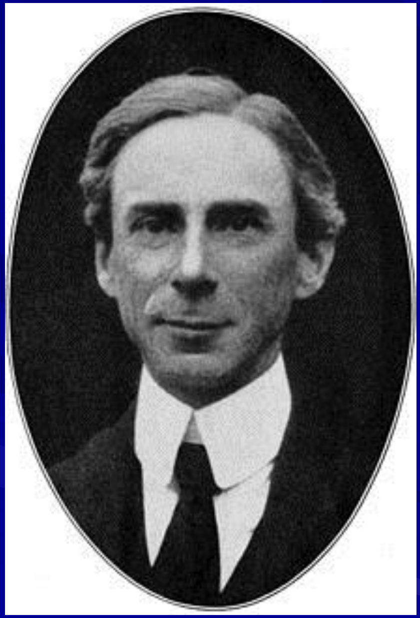
Lecture Note

Mathematics

Logic is the childhood of mathematics and mathematics is the manhood of logic.

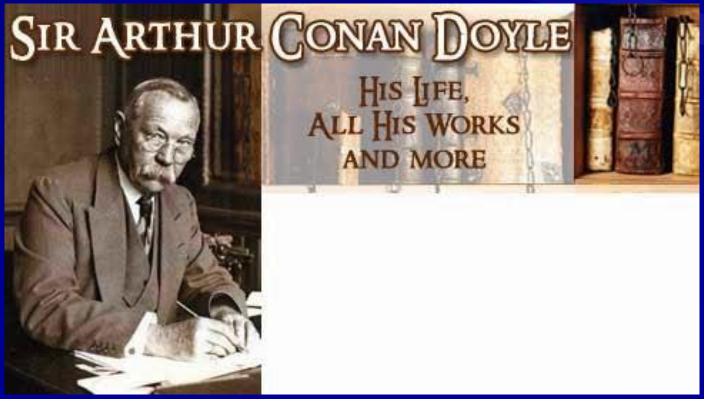
Bertrand Russell



Mathematics

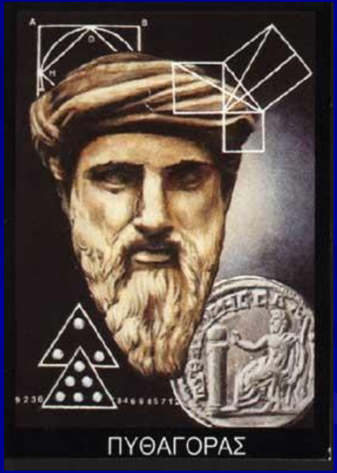
From the drop of water, a logician could infer the possibility of an Atalantic Ocean or Nigara Fall without having seen or heard of the one or the other.

Arthur Conan Doyle : sherlock Holmes



Number is the guide and master of human thought. Without its power, everything would remain obsure and confused

Pythagora



Statement

1 . Statements may be sentences (or) equations (or) inequations (or) identities.

2 . A statement may be true or false , but not both at the same time.

3 . The opposite of false is true and opposite of true is false.

Logical Connectives

$\wedge$ *and* , *Conjunction*

$\vee$ *or* , *Disjunction*

$\Rightarrow$ *implies* , *Conditional*

$\Leftrightarrow$ *if and only if* , *Biconditional*

$:$, $|$ *such that*

$\sim$, *not* , *opposite* , *negation*

$\equiv$, *Equivalence*

Truth Table

P	q	$P \wedge q$	$P \vee q$	$P \Rightarrow q$	$P \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	T

Truth Table

P	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim(P \Leftrightarrow q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

Let p be “ He is tall ” and q be “ He is handsome ” . Write each of the following statements in symbolic form using p and q and find its truth value by using table.

(i) He is tall but not handsome.

(ii) It is false that he is short or handsome

(iii) He is neither tall nor handsome

(iv) He is tall, or he is short and handsome

(v) It is trute that he is short or not handsome

$(i)\ p \wedge \sim q \quad (ii) \sim(\sim p \vee q) \quad (iii) \sim p \wedge \sim q\ (or) \sim(p \vee q)$

$(iv)\ p \vee (\sim p \wedge q) \quad (v) \sim p \vee \sim q\ (or) \sim(p \wedge q)$

$(i)\ p \wedge \sim q$

P	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

$(ii) \sim (\sim p \vee q)$

P	q	$\sim P$	$\sim p \vee q$	$\sim(\sim p \vee q)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

$(iii) \sim p \wedge \sim q$

P	q	$\sim P$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$(iv)\ p \vee (\sim p \wedge q)$

P	q	$\sim P$	$(\sim p \wedge q)$	$p \vee (\sim p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

$(v) \sim p \vee \sim q$

P	q	$\sim P$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Determine the truth value of each of the following composite statements.

(i) If 3 + 2 = 7, then 4+4 =8

p : 3 + 2 = 7, q : 4+4 =8

$$p \Rightarrow q$$

p	q	$p \Rightarrow q$
F	T	T

(ii) It is not true that 2 + 2 = 5, if and only if 4+4 =10

p : 2 + 2 = 5, q : 4+4 =10

$\sim (p \Leftrightarrow q)$

p	q	$p \Leftrightarrow q$	$\sim (p \Leftrightarrow q)$
F	F	T	F

(iii) Yangon is in India or Tokyo is in Myanmar

p : Yangon is in India , q : Tokyo is in Myanmar

$(p \vee q)$

p	q	$p \vee q$
F	F	F

(iv) It is not true that $1 + 1 = 3$ or $2 + 1 = 3$

p : $1 + 1 = 3$, q : $2+1=3$

$\sim (p \vee q)$

p	q	$p \vee q$	$\sim (p \vee q)$
F	T	T	F

(v) It is false that Yangon is in Japan then Paris is in Myanmar

p : Yangon is in Japan , q : Paris is in Myanmar

$\sim (p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\sim (p \Rightarrow q)$
F	F	T	F

Propostion and Truth Table

$$(p \wedge q) \Rightarrow (p \vee q)$$

P	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$(iv)\ (\sim p \vee q) \wedge \sim r$

P	q	r	$\sim p$	$(\sim p \vee q)$	$\sim r$	$(\sim p \vee q) \wedge \sim r$
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

$$(p \Rightarrow q) \vee \sim(p \Leftrightarrow \sim q)$$

P	q	$p \Rightarrow q$	$\sim q$	$(p \Leftrightarrow \sim q)$	$\sim(p \Leftrightarrow \sim q)$	$(p \Rightarrow q) \vee \sim(p \Leftrightarrow \sim q)$
T	T	T	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	T
F	F	T	T	F	T	T

Logical Equivalence

Two propositions P (p , q , r ,...) and Q (p , q , r ,....) are said to be logically equivalent if their truth tables are indentical. We denote this by $P \equiv Q$

Verify that $p \Rightarrow \sim q \equiv q \Rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \Rightarrow \sim q$	$q \Rightarrow \sim p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T
				*	*

By the * *columns* $p \Rightarrow \sim q \equiv q \Rightarrow \sim p$

$$Verify: \quad p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F
						*	*

By the * columns, they are equivalent

$Verify\ :\ \sim(p \Leftrightarrow q) \equiv (p \Leftrightarrow \sim q) \equiv (q \Leftrightarrow \sim p)$

p	q	$\sim p$	$\sim q$	$p \Leftrightarrow q$	$\sim(p \Leftrightarrow q)$	$p \Leftrightarrow \sim q$	$q \Leftrightarrow \sim p$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	F
					*	*	*

By the * columns, they are equivalent

Validity of Argument

A statement that a set of propositions $P_1, P_2, P_3, \ldots, P_n$ yields another proposition P is called an argument on propositions.

The propositions $P_1, P_2, P_3, \ldots, P_n$ are called premises and the proposition P is called concusion.

Argument form is shown as follows;

$$\left.\begin{array}{c} P_1(p, q, r, \ldots) \\ P_2(p, q, r, \ldots) \\ . \\ . \\ . \\ P_n(p, q, r, \ldots) \end{array}\right\} \quad \textit{premises}$$

$$\overline{P(p, q, r, \ldots)} \qquad \textit{Conclusion}$$

Since an argument is also a statement it has a true value.

An argument on propositions is said to be valid if the conclusion P is the true whenver the premises $P_1, P_2, P_3, \ldots\ldots, P_n$ are true

Example:

If everyone does not have love, he is lonely. A person who fells loneliness can early die. Therefore, if he has no love, he can early die.

Let p be " A person who does not have love "

q be " A person who is lonly "

r be " A person who can early die "

$$\left.\begin{array}{c} p \Rightarrow q \\ q \Rightarrow r \end{array}\right\} \quad premises$$

$$\therefore\ p \Rightarrow r \qquad Conclusion$$

			P_1	P_2	P
p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

In 1st, 5th, 7th and 8th rows, premises are true then conclusions are true. Therefore, this argument is valid.

Test the validity of the argument. If it rain, then Mg Mya will be sick. It did not rain. Therefore Mg Mya was not sick.

Solution:

Let p be " It rain "

q be " Mg Mya is sick "

$$\left.\begin{array}{r} p \Rightarrow q \\ \sim p \end{array}\right\} \quad \textit{premises}$$

$$\frac{}{\sim q} \quad \textit{Conclusion}$$

p	q	$p \Rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

*

In 4h row, premises is true then conclusions is false. Therefore, this argument is not valid.

$$\begin{array}{ll} \left.\begin{array}{c} p \Rightarrow q \\ r \Rightarrow \sim q \end{array}\right\} & \textit{premises} \\ \hline \therefore r \Rightarrow \sim p & \textit{Conclusion} \end{array}$$

					P_1	P_2	P	
p	q	r	$\sim p$	$\sim q$	$p \Rightarrow q$	$r \Rightarrow \sim q$	$r \Rightarrow \sim p$	
T	T	T	F	F	T	F	F	
T	T	F	F	F	T	T	T	*
T	F	T	F	T	F	T	T	
T	F	F	F	T	F	T	T	
F	T	T	T	F	T	F	T	
F	T	F	T	F	T	T	T	*
F	F	T	T	T	T	T	T	*
F	F	F	T	T	T	T	T	*

In 2nd ,6th , 7th and 8th rows, premises are true then conclusions are true. Therefore, this argument is valid.

If Ma Aye gets married, then either Ma Hla is maid of honour or Ma Mya maid of honour. If Ma Hla is maid of honour and Ma Mya is maid of honour, then there will be a quarrel at the wedding. Therefore if Ma Aye gets married, then there will be quarrel at the wedding.

p : Ma Aye gets married

q : Ma Hla is maid of honour

r : Ma Mya maid of honour

s : there is a quarrel at the wedding

$$\left.\begin{array}{l} p \Rightarrow q \vee r \\ q \wedge r \Rightarrow s \end{array}\right\} \quad \textit{premises}$$

$$\therefore\ p \Rightarrow s \qquad \textit{Conclusion}$$

p	q	r	s	$q \vee r$	$q \wedge r$	$p \Rightarrow (q \vee r)$	$(q \wedge r) \Rightarrow s$	$p \Rightarrow s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	F
T	T	F	T	T	F	T	T	T
T	T	F	F	T	F	T	T	F
T	F	T	T	T	F	T	T	T
T	F	T	F	T	F	T	T	F
T	F	F	T	F	F	F	T	T
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	F	T
F	T	F	T	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	T	F	T	F	T	T	T
F	F	F	T	F	F	T	T	T
F	F	F	F	F	F	T	T	T

*

In 4^{h} row, premises is true then conclusions is false. Therefore, this argument is not valid.

In 4^{h} and 6^{th} rows, premises are true then conclusions are false. Therefore, this argument is not valid.

Quantifiers

Universal Quantifier

$$(\forall\ x \in U\ )\, p(x) \quad or \quad \forall\, x, p(x)$$

$$If\ \{x /\ \ x \in U\ )\, p(x)\} = U\ ,\ then\ \forall\, x, p(x)\ is\ true$$

$$If\ \{x /\ \ x \in U\ )\, p(x)\} \neq U\ ,\ then\ \forall\, x, p(x)\ is\ flase$$

Existential Quantifier

$$(\exists x \in U)\, p(x) \quad or \quad \exists x, p(x)$$

$$If\ \{x /\ \ x \in U)\, p(x)\} \neq \phi\ , then\ \exists x, p(x)\ is\ true$$

$$If\ \{x /\ \ x \in U)\, p(x)\} = \phi\ , then\ \exists x, p(x)\ is\ flase$$

Negation of Proposition Which contain quantifiers

Let p (x) be a statement. The equivalent form of a negation of a statement which contain quantifiers is given by the following theorem.

$$(1)\ \sim(\forall x \in U)\, p(x)\} \equiv (\exists x \in U) \sim p(x)$$

$$(2)\ \sim(\exists x \in U)\, p(x) \equiv (\forall x \in U) \sim p(x)$$

Determine the truth value of each of the following. Here the universal set U = R

$(i)\ \forall x, |x| = x$

$Since\ , |-1| \neq -1$

$T_p \neq R$

Therefore, it is false

$(ii)\ \exists x, x^2 = x$

$Since\ , 1^2 = 1$

$T_p \neq \phi$

Therefore , it is true

$(iii)\ \forall x, x+1 > x$

$$T_p = R$$

Therefore, it is true

$(iv)\ \exists x, |x| = 0$

$Since\ , |0| = 0$

$$T_p \neq \phi$$

Therefore, it is true

$(v)\ \exists x, x + 2 = x$

$$T_p = \phi$$

Therefore , it is false

$(vi)\ \forall x, x + 2 > 5$

$Since\ , 1 + 2 < 5$

$$T_p \neq R$$

Therefore, it is false

Prove that the following statements are false. Here U = R = the set of real numbers.

$(i)\ \forall x,\ x^2 - 3x + 2 = 0$

$$T_p = \{1, 2\} \neq R$$

Therefore, it is false

$(ii)\ \forall x,\ 5x + 1 = 2x + 9$

$$T_p = \{\frac{8}{3}\} \neq R$$

Therefore, it is false

$(iii)\ \forall x,\ x^2 + 9 = (x + 3)(x - 3)$

$$T_p = \phi \neq R$$

Therefore, it is false

$(iv)\ \exists x,\ x^2 - 4 \neq (x + 2)(x - 2)$

$$T_p = \phi$$

Therefore, it is false

$(v)\ \exists x,\ x \neq x$

$$T_p = \phi$$

Therefore, it is false

Find the counter example for each of the following statement where A = { 2 , 3 , 4 , 5 , 6 , 7 ,8 , 9 }

$(i)\ \forall x \in A,\ x + 5 < 12$

$Take\ \ 9 \in A,$

$then\ \ 9 + 5 > 12\ \ is\ \ false$

Therefore , this statement is false

$(ii)\ \forall x \in A,\ x\ is\ prime$

$Take\ \ 9 \in A,$

$then\ \ 9\ is\ not\ prime$

Therefore, this statement is false

$(iii)\ \forall x \in A,\ x\ is\ even$

$Take\ \ 9 \in A,$

$then\ \ 9\ \ is\ not\ even$

Therefore , this statement is false

$(iv)\ \forall x \in A,\ |x - 4| < 3$

$Take\ \ 9 \in A,$

$then\ \ |9 - 4| > 3$

Therefore , this statement is false

$(i)\ \forall x \in A,\ x^2 - 1 > 7$

$Take\ \ 2 \in A,$

$then\ \ 2^2 - 4 < 7$

Therefore , this statement is false

Negate each of the statement .

$(i)\ \forall x, |x| = x$

$$\sim(\forall x, |x| = x) \equiv \exists x, |x| \neq x$$

$(ii)\ \exists x, x^2 = x$

$$\sim(\exists x, x^2 = x) \equiv \forall x, x^2 \neq x$$

$(\mathrm{iii})\ \forall \mathrm{x}, \mathrm{x}+1 > \mathrm{x}$

$$\sim(\forall \mathrm{x}, \mathrm{x}+1 > \mathrm{x}) \equiv \exists \mathrm{x}, \mathrm{x}+1 \leq \mathrm{x}$$

$(iv)\ \exists x, |x| = 0$

$$\sim(\exists x, |x| = 0) \equiv \forall \mathrm{x}, |\mathrm{x}| \neq 0$$

$$(v)\ \exists x, x + 2 = x$$

$$\sim(\exists x, x + 2 = x) \equiv \forall x, x + 2 \neq x$$

$$(vi)\ \forall x, x + 2 > 5$$

$$\sim(\forall x, x + 2 > 5) \equiv \exists x, x + 2 \leq 5$$

(i) Fill the, $\sim(\exists \Delta ABC, AB + BC \leq AC) \equiv$ ________________

(ii) Fill the $\rightarrow$ or $\leftrightarrow$: $x = 0$ and $y = 0$ ______ $xy = 0$

(iii) Fill the $\rightarrow$ or $\leftrightarrow$: $y = x^2$ ______ $\frac{dy}{dx} = 2x$

(iv) Fill the $\forall$ or $\exists$: ___ $\Delta ABC, \angle A + \angle B = \angle C$

(v) Fill the $\forall$ or $\exists$: __ $\Delta ABC : \angle A + \angle B + \angle C = 180$

State the Pythagoras Theorem with logical notations

$$\forall \Delta ABC : \angle C = 90^{\circ} \Leftrightarrow \quad AB^2 = AC^2 + BC^2$$

Re-write the statement " a divides b if and only if some integer n, b = na " by using logic notations.

$$a \text{ divides } b \Leftrightarrow \exists n \in J : b = n\,a$$