Differentiation

$$* \quad \frac{d}{dx} k\, f(x) = k \frac{d}{dx} f(x)$$

$$* \quad \frac{d}{dx} k\, x = k$$

$$* \quad \frac{d}{dx}[f(x) \pm g(x)]$$

$$= \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

$$1* \frac{d}{dx} x^n = n\, x^{n-1}$$

Integration

$$* \quad \int k\, f(x)\, dx = k \int f(x)\, dx$$

$$* \quad \int k\, dx = k \int dx = k\, x + C$$

$$* \quad \int [f(x) \pm g(x)]\, dx$$

$$= \int f(x)\, dx \pm \int g(x)\, dx$$

$$1 * \quad \int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad , \; n \neq -1$$

$$\int (a\,x + b)^n\, dx = \frac{(a\,x+b)^{n+1}}{(n+1)\,a} + C$$

$$\int [f(x)]^n \; f'(x)\, dx = \frac{[f(x)]^{n+1}}{(n+1)} + C$$

$$e.g* \quad \int 3x^6\,dx = 3\frac{x^7}{7} + C$$

$$* \quad \int (1-3x)^{-\frac{1}{3}}\,dx = \frac{(1-3x)^{\frac{2}{3}}}{\frac{2}{3}(-3)} + C = -\frac{1}{2}(1-3x)^{\frac{2}{3}} + C$$

$$* \quad \int \left(1-2x^3\right)^6 x^2 dx = -\tfrac{1}{6} \int \left(1-2x^3\right)^6 (-6x^2)\,dx$$

f (x)^6 f'(x)

$$= -\frac{1}{6}\,\frac{\left(1-2x^3\right)^7}{7} + C$$

$$2 * \quad \frac{d}{dx} e^{x} = e^{x}$$

$$2 * \quad \int e^{x}\, dx = e^{x} + C$$

$$\int e^{ax+b}\, dx = \frac{e^{ax+b}}{a} + C$$

$$\int e^{f(x)} . f'(x)\, dx = e^{f(x)} + C$$

$$e.g * \quad \int e^{1-3x}\, dx = \frac{e^{1-3x}}{-3} + C$$

$$* \quad \int e^{1-x^{4}} . x^{3}\, dx$$

$$= -\frac{1}{4} \int e^{1-x^{4}} (-4x^{3})\, dx$$

$$= -\frac{1}{4} e^{1-x^{4}} + C$$

$$3 * \quad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$3 * \quad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$* \int \frac{1}{(ax+b)}\,dx = \frac{1}{a}\ln(ax+b)+C$$

$$* \int \frac{f'(x)}{f(x)}\,dx \quad = \ln\big(f(x)\big)+C$$

ပိုင်းခြေ ရှိတ်လိုပိုင်းဝေရေလျှင် ln (ပိုင်းခြေ)

$$* \int \frac{x^2-2}{x^3-6x+13}\,dx \quad = \frac{1}{3}\int \frac{3x^2-6}{x^3-6x+13}\,dx$$

$$= \frac{1}{3}\ln\left(x^3-6x+13\right)+C$$

$$4 * \quad \frac{d}{dx}\sin = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\cos ec^2 x$$

$$\frac{d}{dx}\sec x = \sec x . \tan x$$

$$\frac{d}{dx}\cos ec\, x = -\cos ec\, x . \cot x$$

$$4 * \quad \int \cos x\, dx = \sin x + C$$

$$\int \cos(ax+b)\, dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \cos f(x) . f'(x)\, dx = \sin f(x) + C$$

Similarly

$$\int \sin x\, dx = -\cos x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \cos ec^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \cos ec\, x . \cot x\, dx = -\cos ec\, x + C$$

$$\int \sin(3x+5)\,dx = -\frac{\cos(3x+5)}{3} + C$$

$$\int \cos ec^2 5x\,dx = -\frac{\cot 5x}{5} + C$$

$$\int \sec^2(\ln x)\,.\,\frac{1}{x}\,dx = \tan(\ln x) + C$$

$$\int \cos(1-3x^2)\,.\,x\,dx = -\frac{1}{6}\int \cos(1-3x^2)\,.(-6x)\,dx$$

$$= -\frac{1}{6}\sin(1-3x^2) + C$$

$$5 * \quad \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-(x+b)^2}} dx = \sin^{-1}\left(\frac{x+b}{a}\right) + C$$

$$e.g * \int \frac{1}{\sqrt{2-x^2}} dx = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$* \int \frac{1}{\sqrt{7-x^2-6x}} dx = \int \frac{1}{\sqrt{16-(x^2+6x+9)}} dx = \int \frac{1}{\sqrt{16-(x+3)^2}} dx$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

$$5 * \quad \int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{(x+b)^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x+b}{a}\right) + C$$

$$e.g * \int \frac{1}{2+x^2}\,dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$$

$$* \int \frac{x-2}{x^2+6x+13}\,dx = \int \frac{x+3-5}{x^2+6x+13}\,dx$$

$$= \int \frac{x+3}{x^2+6x+13}\,dx - 5\int \frac{1}{x^2+6x+9+4}\,dx$$

$$= \frac{1}{2}\int \frac{2x+6}{x^2+6x+13}\,dx - 5\int \frac{1}{(x+3)^2+4}\,dx$$

$$= \frac{1}{2}\ln(x^2+6x+13) - \frac{5}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + C$$

$* \tan x = \dfrac{\sin x}{\cos x}$

$* \tan^2 x = \sec^2 x - 1$

$* \sin^2 x = 1 - \cos^2 x$

$(or)\frac{1}{2}\left(1 - \cos 2x\right)$

$* \cot x = \dfrac{\cos x}{\sin x}$

$* \cot^2 x = \cos ec^2 x - 1$

$* \cos^2 x = 1 - \sin^2 x$

$(or)\frac{1}{2}\left(1 + \cos 2x\right)$

$* \; A > B$

$$\sin A.\cos B = \frac{1}{2}\left(\sin(A+B)+\sin(A-B)\right)$$

$$\cos A.\sin B = \frac{1}{2}\left(\sin(A+B)-\sin(A-B)\right)$$

$$\sin A.\sin B = \frac{1}{2}\left(\cos(A-B)-\cos(A+B)\right)$$

$$\cos A.\cos B = \frac{1}{2}\left(\cos(A-B)+\cos(A+B)\right)$$

$* \sin 2A = 2\sin A.\cos A$

$$e.g*\int \tan 3x\,dx = \int \frac{\sin 3x}{\cos 3x}dx = -\frac{1}{3}\int \frac{-3\sin 3x}{\cos 3x}dx = -\frac{1}{3}\ln(\cos 3x) + C$$

$$e.g*\int \sin^2\left(\frac{1-x}{3}\right)dx = \frac{1}{2}\int\left(1-\cos\frac{2-2x}{3}\right)dx = \frac{1}{2}\left[x - \frac{\sin\left(\frac{2-2x}{3}\right)}{-\frac{2}{3}}\right] + C$$

$$e.g*\int \sin^3 3x\,dx = \int \sin^2 3x.\sin 3x\,dx = \int\left(1-\cos^2 3x\right).\sin 3x\,dx$$

$$= \int\left(\sin 3x - \cos^2 3x.\sin 3x\right)dx = \int\left(\sin 3x + \tfrac{1}{3}\cos^2 3x.(-3\sin 3x)\right)dx$$

$(f (x))^2$ f'(x)

$$= -\frac{\cos 3x}{3} + \frac{1}{3}\,\frac{\cos^3 3x}{3} + C$$

$$e.g*\int \cos^2(1-3x)\,dx = \frac{1}{2}\int\left(1+\cos(2-6x)\right)dx = \frac{1}{2}\left[x + \frac{\sin(2-6x)}{-6}\right] + C$$

$$e.g*\int \cos^5 x\, d x = \int \left(\cos^2 x\right)^2 .\cos x\, d x \quad = \int \left(1-\sin^2 x\right)^2 .\cos x\, d x$$

$$= \int \left(1-2\sin^2 x+\sin^4 x\right).\cos x\, d x$$

$$= \int \left(\cos x-2\sin^2 x\cos\, x+\sin^4 x\cos x\right)\, d x$$

$$= \sin x\, -2\,\frac{\sin^3\, x}{3} \; + \frac{\sin^5\, x}{5} + C$$

$$e.g*\int \sin 3x.\cos 5x\, d x \quad = \frac{1}{2}\int (\sin 8x-\sin 2x)\, d x$$

$$= \frac{1}{2}\left[-\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right]+C$$

$$e.g*\int \cos 3x.\cos 5x\, d x \; = \frac{1}{2}\int (\cos 2x+\cos 8x)\, d x$$

$$= \frac{1}{2}\left[\frac{\sin 2x}{2}+\frac{\sin 8x}{8}\right]+C$$

Substitution

$$*\int \frac{1}{x^2} \tan^2(\frac{1}{x})dx$$

$$t = \frac{1}{x}$$

$$\frac{dt}{dx} = -\frac{1}{x^2}$$

$$-dt = \frac{1}{x^2}dx$$

$$\int \frac{1}{x^2} \tan^2(\frac{1}{x})dx \quad = -\int \tan^2 t\,dt$$

$$= -\int (\sec^2 t - 1)\,dt$$

$$= \int (1 - \sec^2 t)\,dt$$

$$= t - \tan t + C$$

$$= \frac{1}{x} - \tan(\frac{1}{x}) + C$$

$$*\int \frac{1}{x} \cos^2 (1-4\ln x) dx$$

$$t = 1-4\ln x$$

$$\frac{dt}{dx} = -4\frac{1}{x}$$

$$-\frac{1}{4} dt = \frac{1}{x} dx$$

$$\int \frac{1}{x} \cos^2 (1-4\ln x) dx = -\frac{1}{4}\int \cos^2 t \, dt$$

$$= -\frac{1}{8}\int (1+\cos 2t) \, dt$$

$$= -\frac{1}{8}\left(t + \frac{\sin 2t}{2} \right) + C$$

$$= -\frac{1}{8}\left((1-4\ln x) + \frac{\sin 2(1-4\ln x)}{2} \right) + C$$

$$*\int \frac{1}{x}\tan(2-3\ln x)dx$$

$$t=2-3\ln x$$

$$\frac{dt}{dx}=-3\frac{1}{x}$$

$$-\frac{1}{3}dt=\frac{1}{x}dx$$

$$\int \frac{1}{x}\tan(2-3\ln x)dx = -\frac{1}{3}\int \tan t\ dt$$

$$= -\frac{1}{3}\int \frac{\sin t}{\cos t}\ dt$$

$$= \frac{1}{3}\int \frac{-\sin t}{\cos t}\ dt$$

$$= \frac{1}{3}\ln(\cos t) + C$$

$$= \frac{1}{3}\ln(\cos(2-3\ln x))+ C$$

Exercises

$$1*\int\left(\frac{1}{x^{2}}-\frac{3}{x^{1/3}}+\frac{5}{x}-x^{3/2}\right)dx=\frac{x^{-1}}{-1}-3\frac{x^{2/3}}{2/3}+5\ln x-\frac{x^{5/2}}{5/2}+C$$

$$2*\int\left(\frac{x^{3}-x+7}{\sqrt{x}}\right)dx=\int\left(x^{\frac{5}{2}}-x^{\frac{1}{2}}+7x^{-\frac{1}{2}}\right)dx=\frac{x^{\frac{7}{2}}}{\frac{7}{2}}-\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+7\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$3*\int\left(x^{-\frac{1}{5}}+\frac{7}{3\sqrt{x}}\right)dx=\frac{x^{\frac{4}{5}}}{\frac{4}{5}}+\frac{7}{3}\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$4*\int\left(\frac{4x-5e^{x}}{xe^{x}}\right)dx=\int\left(4e^{-x}-\frac{5}{x}\right)dx=\frac{4e^{-x}}{-1}-5\ln x+C$$

$$5*\int\frac{x^{3}}{1-x^{4}}dx=-\frac{1}{4}\int\frac{-4x^{3}}{1-x^{4}}dx=-\frac{1}{4}\ln(1-x^{4})+C$$

$$6*\int\frac{\sin 3x}{\cos^{2}3x}dx=\int\tan 3x\sec 3x\,dx=\frac{\sec 3x}{3}+C$$

$$7*\int\left(\tan^{2}3x-6\right)dx=\int\left(\sec^{2}3x-7\right)dx=\frac{\tan 3x}{3}-7x+C$$

$$8*\int\frac{5}{x\ln x}dx=5\int\frac{\frac{1}{x}}{\ln x}dx=5\ln\left(\ln x\right)+C$$

$$9*\int\frac{3}{x\ln x^{2}}dx=\frac{3}{2}\int\frac{\frac{2}{x}}{\ln x^{2}}dx=\frac{3}{2}\ln\left(\ln x^{2}\right)+C$$

$$10*\int\sec 3x\,dx=\int\sec 3x\times\frac{\sec 3x+\tan 3x}{\sec 3x+\tan 3x}dx=\int\frac{\sec^{2}3x+\sec 3x.\tan 3x}{\sec 3x+\tan 3x}dx$$

$$=\frac{1}{3}\int\frac{3\left(\sec^{2}3x+\sec 3x.\tan 3x\right)}{\tan 3x+\sec 3x}dx=\frac{1}{3}\ln\left(\tan 3x+\sec 3x\right)+C$$

$$11*\int\cos ec\,2x\,dx=\int\cos ec\,2x\times\frac{\cos ec\,2x+\cot 2x}{\sec 2x+\tan 2x}dx$$

$$=\int\frac{\cos ec^{2}\,2x+\cos ec\,2x.\cot 2x}{\cos ec\,2x+\cot 2x}dx=-\frac{1}{2}\int\frac{-2\left(\cos ec^{2}\,2x+\cos ec\,2x.\cot 2x\right)}{\cot 2x+\cos ec\,2x}dx$$

$$=-\frac{1}{2}\ln\left(\cot 2x+\cos ec\,2x\right)+C$$

$$12 * \int \frac{x^2-2x+1}{x^2+3}\,dx = \int \left(1-\frac{2x}{x^2+3}-2\frac{1}{x^2+3}\right)dx$$

$$\begin{array}{r} 1 \\ x^2+3 \,\overline{)\, x^2-2x+1} \\ x^2 \qquad\quad +3 \\ \hline -2x-2 \end{array}$$

$$= x - \ln\left(x^2+3\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$13 * \int \frac{3}{x^{\frac{2}{3}}(2-x^{\frac{1}{3}})^6}\,dx = 3\int x^{-\frac{2}{3}}(2-x^{\frac{1}{3}})^{-6}\,dx = -9\int (2-x^{\frac{1}{3}})^{-6}(-\tfrac{1}{3}x^{-\frac{2}{3}})\,dx$$

$$= \frac{9}{5}\left(2-x^{\frac{1}{3}}\right)^{-5} + C$$

$$14*\int \frac{3}{2+e^{-x}}\,dx = \int\left(\frac{3}{2}-\frac{3}{2}\,\frac{e^{-x}}{(2+e^{-x})}\right)dx = \int\left(\frac{3}{2}+\frac{3}{2}\,\frac{-e^{-x}}{(2+e^{-x})}\right)dx$$

$$\begin{array}{r|l} & \frac{3}{2} \\ \hline 2+e^{-x} & 3 \\ & 3+\frac{3}{2}e^{-x} \\ \hline & -\frac{3}{2}e^{-x} \end{array}$$

$$=\frac{3}{2}x+\frac{3}{2}\ln(2+e^{-x})+C$$

$$15*\int \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\,dx = \ln\,(e^{x}-e^{-x})+C$$

$$\frac{d}{dx}(e^{x}-e^{-x}) = (e^{x}+e^{-x})$$

$$16*\int \frac{\cos 3x}{5-\sin 3x}\,dx = -\frac{1}{3}\int \frac{-3\cos 3x}{5-\sin 3x}\,dx$$

$$=-\frac{1}{3}\ln\,(5-\sin 3x)+C$$

$$\frac{d}{dx}(5-\sin 3x) = -3\cos 3x$$

$$17*\int \frac{\tan 2x}{\ln(\cos 2x)}\,dx = -\frac{1}{2}\int \frac{-2\tan 2x}{\ln(\cos 2x)}\,dx$$

$$=-\frac{1}{2}\ln\,(\ln(\cos 2x))+C$$

$$\frac{d}{dx}\ln(\cos 2x)$$

$$=\frac{1}{\cos 2x}(-\sin 2x).2$$

$$18*\int \frac{1-\tan 2x}{1+\tan 2x}\,dx = \int \frac{1-\dfrac{\sin 2x}{\cos 2x}}{1+\dfrac{\sin 2x}{\cos 2x}}\,dx = \int \frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}\,dx$$

$$= \frac{1}{2}\int \frac{2\cos 2x - 2\sin 2x}{\cos 2x+\sin 2x}\,dx = \frac{1}{2}\ln(\cos 2x+\sin 2x)+C$$

$$19*\int \sec^4 x\,dx = \int \sec^2 x.\sec^2 x\,dx = \int (1+\tan^2 x).\sec^2 x\,dx$$

$$= \int (\sec^2 x+\tan^2 x.\sec^2 x)\,dx = \tan x + \frac{\tan^3 x}{3} + C$$

$$20*\int \tan^4 x\,dx = \int \tan^2 x.\tan^2 x\,dx = \int (\sec^2 x-1).\tan^2 x\,dx$$

$$= \int (\tan^2 x.\sec^2 x - \tan^2 x)\,dx = \int (\tan^2 x.\sec^2 x - \sec^2 x + 1)\,dx$$

$$= \frac{\tan^3 x}{3} - \tan x + x + C$$

$$21*\int \sin 2x.\cos 5x\,dx = \frac{1}{2}\int(\sin 7x - \sin 3x)\,dx = \frac{1}{2}\left(-\frac{\cos 7x}{7} + \frac{\cos 3x}{3}\right) + C$$

$$22*\int \sin 3x.\sin 5x\,dx = \frac{1}{2}\int(\cos 2x - \cos 8x)\,dx = \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right) + C$$

$$23*\int \frac{(1+\sqrt{x})^2}{\sqrt[3]{x}}\,dx = \int \frac{1+2\sqrt{x}+x}{\sqrt[3]{x}}\,dx = \int\left(x^{-\frac{1}{3}} + 2x^{\frac{1}{6}} + x^{\frac{2}{3}}\right)dx$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 2\frac{x^{\frac{7}{6}}}{\frac{7}{6}} + \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C$$

$$24*\int \sec 2x(\sec 2x - \sin 3x.\cos 2x)\,dx = \int(\sec^2 2x - \sin 3x)\,dx$$

$$= \frac{\tan 2x}{2} + \frac{\cos 3x}{3} + C$$

$$25 * \int \frac{2+\sqrt{x}}{2-\sqrt{x}}\, d\, x$$

$$= \int \left(-1 - 4\,x^{-\frac{1}{2}} + 8\,\frac{x^{-\frac{1}{2}}}{(2-\sqrt{x}\,)} \right) d\, x$$

$$= \int \left(-1 - 4\,x^{-\frac{1}{2}} - 16\,\frac{-\frac{1}{2}\,x^{-\frac{1}{2}}}{(2-\sqrt{x}\,)} \right) d\, x$$

$$= x \quad - 4\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \quad - 16\,\ln(2-\sqrt{x}\,) + C$$

$$\begin{array}{r|l} & -1 - 4\,x^{-\frac{1}{2}} \\ \hline -\sqrt{x} + 2 & \sqrt{x} + 2 \\ & \sqrt{x} - 2 \\ \hline & 4 \\ & 4 - 8\,x^{-\frac{1}{2}} \\ \hline & \qquad +8\,x^{-\frac{1}{2}} \end{array}$$

$$\frac{d}{d\,x}(2-\sqrt{x}\,) = -\frac{1}{2}\,x^{-\frac{1}{2}}$$

$$26*\int \frac{1}{1+\sqrt{2x}}\,dx$$

$$=\int\left((2x)^{-\frac{1}{2}}-\frac{(2x)^{-\frac{1}{2}}}{(1+\sqrt{2x})}\right)dx$$

$$=\frac{(2x)^{\frac{1}{2}}}{\frac{1}{2}\times 2}-\ln(1+\sqrt{2x})+C$$

$$\begin{array}{r l} & (2x)^{-\frac{1}{2}} \\ \sqrt{2x}+1 & \overline{)\,1\qquad\qquad} \\ & \underline{1+(2x)^{-\frac{1}{2}}} \\ & -(2x)^{-\frac{1}{2}} \end{array}$$

$$\frac{d}{dx}(1+\sqrt{2x})$$

$$=\frac{1}{2}(2x)^{-\frac{1}{2}}.2$$

Integration by parts

$$*\int u.dv = u.v - \int v.du$$

$$*\int \left(f(x).g(x) \right) dx$$

$$f'(x) \neq g(x) \quad and \quad g'(x) \neq f(x)$$

$$Possible\ ,\ u = f(x) \quad , dv = g(x)dx$$

$$\frac{du}{dx} = f'(x)\ ,\int dv = \int g(x)dx$$

$$du = f'(x)dx \quad , v = G(x)$$

Selection of u and dv

$$1 * \int \begin{matrix} x^{p} \\ (ax+b) \end{matrix} \; . \begin{matrix} \ln \\ \tan^{-1} \\ \sin^{-1} \end{matrix} \qquad dx$$

$$u = \begin{matrix} \ln \\ \tan^{-1} \\ \sin^{-1} \end{matrix} \qquad , \; dv = \begin{matrix} x^{p} \\ (ax+b) \end{matrix} \qquad dx$$

$$2 * \int x^{p} \quad . e \qquad dx$$

$(ax+b)$

$\sin$

$\cos$

$\sec^{2}$

$-$

$-$

$(cx+d)^{n}$

$, \; dv \; = e \qquad dx$

$u = x^{p}$

$(ax+b)$

$\sin$

$\cos$

$\sec^{2}$

$-$

$-$

$(cx+d)^{n}$

$$3 * \int e \ . \sin \qquad d x$$

$$\cos$$

Any u and dv

$$1^{*}\int \ln(1-2x)\,dx$$

$$u=\ln(1-2x) \quad , \quad dv=dx$$

$$\frac{du}{dx}=\frac{1}{(1-2x)}(-2) \quad , \quad \int dv=\int dx$$

$$du=\frac{2}{(2x-1)}dx \quad , \quad v=x$$

$$\begin{array}{r|l} & \quad 1 \\ \hline 2x-1 & 2x \\ & 2x-1 \\ \hline & \qquad 1 \end{array}$$

$$^{*}\int u.dv=u.v-\int v.du$$

$$\int \ln(1-2x)\,dx=\ln(1-2x).\ x-\int x.\frac{2}{(2x-1)}dx$$

$$=\ln(1-2x).\ x-\int\left(1+\frac{1}{2x-1}\right)dx$$

$$=\ln(1-2x).\ x-x-\frac{1}{2}\ln(2x-1)+C$$

$$2*\int \tan^{-1} 2x\,dx$$

$$u = \tan^{-1} 2x \qquad , \quad dv = dx$$

$$\frac{du}{dx} = \frac{1}{4x^2+1}(2) \qquad , \quad \int dv = \int dx$$

$$du = \frac{2}{4x^2+1}dx \qquad , \quad v = x$$

$$*\int u.dv = u.v - \int v.du$$

$$\int \tan^{-1} 2x\,dx = \tan^{-1} 2x \cdot x - \int x.\frac{2}{4x^2+1}dx$$

$$= \tan^{-1} 2x \,.\, x - \frac{1}{4}\int \frac{8x}{4x^2+1}dx$$

$$= \tan^{-1} 2x \,.\, x - \frac{1}{4}\ln(4x^2+1) + C$$

$$3 * \int x^2 \tan^{-1} x^3 \, dx$$

$$u = \tan^{-1} x^3 \quad , \quad dv = x^2 \, dx$$

$$\frac{du}{dx} = \frac{1}{x^6+1}(3x^2) \; , \quad \int dv = \int x^2 \, dx$$

$$du = \frac{3x^2}{x^6+1} dx \quad , \quad v = \frac{x^3}{3}$$

$$* \int u \, . \, dv = u \, . \, v - \int v \, . \, du$$

$$\int x^2 \tan^{-1} x^3 \, dx = \tan^{-1} x^3 . \frac{x^3}{3} - \int \frac{x^3}{3} . \frac{3x^2}{x^6+1} dx$$

$$= \tan^{-1} x^3 \; . \frac{x^3}{3} - \frac{1}{6} \int \frac{6x^5}{x^6+1} dx$$

$$= \tan^{-1} x^3 . \frac{x^3}{3} - \frac{1}{6} \ln(x^6+1) + C$$

$$4 * \int x . \tan^{-1} x \, d x$$

$$u = \tan^{-1} x \quad , \quad d v = x \, d x$$

$$\frac{d u}{d x} = \frac{1}{x^{2} + 1} \quad , \quad \int d v = \int x \, d x$$

$$d u = \frac{1}{x^{2} + 1} d x \quad , \quad v = \frac{x^{2}}{2}$$

$$\begin{array}{r|l} & 1 \\ \hline x^{2} + 1 & x^{2} \\ & x^{2} + 1 \\ \hline & -1 \end{array}$$

$$* \int u . d v = u . v - \int v . d u$$

$$\int x . \tan^{-1} x \, d x = \tan^{-1} x . \frac{x^{2}}{2} - \int \frac{x^{2}}{2} . \frac{1}{x^{2} + 1} d x$$

$$= \frac{1}{2} x^{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^{2} + 1} \right) d x$$

$$= \frac{1}{2} x^{2} \tan^{-1} x - \frac{1}{2} \left(x - \tan^{-1} x \right) + C$$

$$5*\int \sin^{-1} x\,dx$$

$$u=\sin^{-1} x \quad , \quad dv = dx$$

$$\frac{du}{dx}=\frac{1}{\sqrt{1-x^2}} \quad , \quad \int dv = \int dx$$

$$du=\frac{1}{\sqrt{1-x^2}}dx \quad , \quad v = x$$

$$*\int u.dv = u.v - \int v.du$$

$$\int \sin^{-1} x\,dx = \sin^{-1} x \;.\, x - \int x.\frac{1}{\sqrt{1-x^2}}dx$$

$$= \sin^{-1} x \;.\, x - \int (1-x^2)^{-\frac{1}{2}}.x\,dx$$

$$= \sin^{-1} x \;.\, x + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}.(-2x)\,dx$$

$$= \sin^{-1} x \;.\, x + \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \sin^{-1} x \;.\, x + \sqrt{1-x^2} + C$$

$$6*\int x.\sin^{-1}x^{2}\,dx$$

$$u=\sin^{-1}x^{2}\qquad , \quad dv=x.dx$$

$$\frac{du}{dx}=\frac{1}{\sqrt{1-x^{4}}}2x\qquad , \quad \int dv=\int x\,dx$$

$$du=\frac{2x}{\sqrt{1-x^{4}}}dx\qquad , \quad v=\frac{x^{2}}{2}$$

$$\int x.\sin^{-1}x^{2}\,dx=\sin^{-1}x^{2}\cdot\frac{x^{2}}{2}-\int\frac{x^{2}}{2}\cdot\frac{2x}{\sqrt{1-x^{4}}}dx$$

$$=\frac{1}{2}x^{2}\sin^{-1}x^{2}+\frac{1}{4}\int(1-x^{4})^{-\frac{1}{2}}(-4x^{3})\,dx$$

$$=\frac{1}{2}x^{2}\sin^{-1}x^{2}+\frac{1}{4}\frac{(1-x^{4})^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{1}{2}x^{2}\sin^{-1}x^{2}+\frac{1}{2}\sqrt{1-x^{4}}+C$$

$$7*\int x e^{1-3x}\,dx$$

$$u = x \qquad , \quad dv = e^{1-3x}.dx$$

$$\frac{du}{dx} = 1 \qquad , \quad \int dv = \int e^{1-3x}\,dx$$

$$du = dx \qquad , \quad v = \frac{e^{1-3x}}{-3}$$

$$\int x e^{1-3x}\,dx = x\,\frac{e^{1-3x}}{-3} - \int \frac{e^{1-3x}}{-3}dx$$

$$= -\frac{1}{3}x.e^{1-3x} + \frac{1}{3}\int e^{1-3x}\,dx$$

$$= -\frac{1}{3}x.e^{1-3x} + \frac{1}{3}\,\frac{e^{1-3x}}{-3} + C$$

$$= -\frac{1}{3}x.e^{1-3x} - \frac{1}{9}\,e^{1-3x} + C$$

$$8^*\int (1-3x)\sin 2x\,dx$$

$$u = (1-3x) \quad , \quad dv = \sin 2x\,dx$$

$$\frac{du}{dx} = -3 \quad , \quad \int dv = \int \sin 2x\,dx$$

$$du = -3\,dx \quad , \quad v = \frac{-\cos 2x}{2}$$

$$\int (1-3x)\sin 2x\,dx = (1-3x)\frac{-\cos 2x}{2} - \int \frac{-\cos 2x}{2}(-3)\,dx$$

$$= \frac{1}{2}(3x-1)\cos 2x - \frac{3}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2}(3x-1)\cos 2x - \frac{3}{4}\sin 2x + C$$

$$9^{*}\int (1-3x)\sec^2 2x\,dx$$

$$u = (1-3x) \quad , \quad dv = \sec^2 2x\,dx$$

$$\frac{du}{dx} = -3 \quad , \quad \int dv = \int \sec^2 2x\,dx$$

$$du = -3\,dx \quad , \quad v = \frac{\tan 2x}{2}$$

$$\int (1-3x)\sec^2 2x\,dx = (1-3x)\,\frac{\tan 2x}{2} - \int \frac{\tan 2x}{2}(-3)\,dx$$

$$= \frac{1}{2}(1-3x).\tan 2x + \frac{3}{2}\int \frac{\sin 2x}{\cos 2x}\,dx$$

$$= \frac{1}{2}(1-3x).\tan 2x - \frac{3}{4}\int \frac{-2\sin 2x}{\cos 2x}\,dx$$

$$= \frac{1}{2}(1-3x).\tan 2x - \frac{3}{4}\ln(\cos 2x) + C$$

$$10*\int (1-3x)^{\frac{2}{3}}(2x+5)\,dx$$

$$u=(2x+5)\quad,\quad dv=(1-3x)^{\frac{2}{3}}\,dx$$

$$\frac{du}{dx}=2\quad,\quad \int dv=\int (1-3x)^{\frac{2}{3}}\,dx$$

$$du=2\,dx\quad,\quad v=\frac{(1-3x)^{\frac{5}{3}}}{\frac{5}{3}(-3)}=-\frac{1}{5}(1-3x)^{\frac{5}{3}}$$

$$\int (1-3x)^{\frac{2}{3}}(2x+5)\,dx=(2x+5)\left(-\frac{1}{5}(1-3x)^{\frac{5}{3}}\right)-\int -\frac{1}{5}(1-3x)^{\frac{5}{3}}(2)\,dx$$

$$=-\frac{1}{5}(2x+5)(1-3x)^{\frac{5}{3}}+\frac{2}{5}\int (1-3x)^{\frac{5}{3}}\,dx$$

$$=-\frac{1}{5}(2x+5)(1-3x)^{\frac{5}{3}}+\frac{2}{5}\,\frac{(1-3x)^{\frac{8}{3}}}{\frac{8}{3}(-3)}$$

$$=-\frac{1}{5}(2x+5)(1-3x)^{\frac{5}{3}}-\frac{1}{20}(1-3x)^{\frac{8}{3}}+C$$

$$11*\int x^2 e^{1-3x}\,dx$$

$$u = x^2 \quad , \quad dv = e^{1-3x}.dx$$

$$\frac{du}{dx} = 2x \quad , \quad \int dv = \int e^{1-3x}\,dx$$

$$du = 2x\,dx \quad , \quad v = \frac{e^{1-3x}}{-3}$$

$$\int x^2 e^{1-3x}\,dx = x^2\,\frac{e^{1-3x}}{-3} - \int \frac{e^{1-3x}}{-3}2x\,dx$$

$$= -\frac{1}{3}x^2.e^{1-3x} + \frac{2}{3}\int e^{1-3x}\,x\,dx$$

$$**\int e^{1-3x}\,x\,dx$$

$$u = x \quad , \quad dv = e^{1-3x}.dx$$

$$\frac{du}{dx} = 1 \quad , \quad \int dv = \int e^{1-3x}\,dx$$

$$du = dx \quad , \quad v = \frac{e^{1-3x}}{-3}$$

$$**\int e^{1-3x}\,x\,dx = x\frac{e^{1-3x}}{-3} - \int -\frac{1}{3}e^{1-3x}\,dx$$

$$= -\frac{1}{3}x\,e^{1-3x} - \frac{1}{9}e^{1-3x}$$

$$\int x^2 e^{1-3x}\,dx = -\frac{1}{3}x^2 . e^{1-3x} - \frac{2}{9}x\,e^{1-3x} - \frac{2}{27}e^{1-3x} + C$$

$$1*\int (3+\tfrac{1}{x})^{\frac{2}{3}} (\tfrac{1}{x^2})\,d\,x = -\int (3+\tfrac{1}{x})^{\frac{2}{3}} (-\tfrac{1}{x^2})\,d\,x = -\frac{(3+\frac{1}{x})^{\frac{5}{3}}}{\frac{5}{3}} + C$$

$$2*\int \sqrt{x}\,(4+x^{\frac{3}{2}})^{\frac{2}{3}}\,d\,x = \frac{2}{3}\int (4+x^{\frac{3}{2}})^{\frac{2}{3}} (\tfrac{3}{2}x^{\frac{1}{2}})\,d\,x = \frac{2}{3}\,\frac{(4+x^{\frac{3}{2}})^{2}}{2} + C$$

$$3*\int \frac{x-1}{(x^2-2x+5)^{\frac{7}{2}}}\,d\,x = \frac{1}{2}\int (x^2-2x+5)^{-\frac{7}{2}}(2x-2)\,d\,x$$

$$= \frac{1}{2}\,\frac{(x^2-2x+5)^{-\frac{5}{2}}}{-\frac{5}{2}} + C = -\frac{1}{5}(x^2-2x+5) + C$$

$$4*\int \frac{e^{2x}}{3+e^{x}}\,d\,x = \int (e^{x} - 3\,\frac{e^{x}}{3+e^{x}})\,d\,x = e^{x} - 3\ln(3+e^{x}) + C$$

$$\begin{array}{r|l} & e^{x} \\ \cline{2-2} e^{x}+3 & e^{2x} \\ & e^{2x}+3e^{x} \\ \cline{2-2} & -3e^{x} \end{array}$$

$$5*\int \tan^2\left(\tfrac{1-2x}{3}\right)dx = \int \left(\sec^2\left(\tfrac{1-2x}{3}\right)-1\right)dx \quad = \frac{\tan\left(\frac{1-2x}{3}\right)}{-\frac{2}{3}} - x + C$$

$$6*\int \frac{\ln(\ln x)}{x}\,dx$$

$$u = \ln(\ln x) \quad , \quad dv = \frac{1}{x}dx$$

$$\frac{du}{dx} = \frac{1}{\ln x}\frac{1}{x} \quad , \quad \int dv = \int \frac{1}{x}dx$$

$$du = \frac{1}{\ln x}\frac{1}{x}dx \quad , \quad v = \ln x$$

$$\int \frac{\ln(\ln x)}{x}dx = \ln(\ln x)\,.\ln x - \int \ln x \frac{1}{\ln x}\frac{1}{x}dx$$

$$= \ln x\;.\ln(\ln x) - \ln x + C$$

$$7*\int x^2.e^{x^3}\cos\left(e^{x^3}\right)dx = \frac{1}{3}\int \cos\left(e^{x^3}\right)\left(3e^{x^3}.x^2\right)dx$$

$$= \frac{1}{3}\sin\left(e^{x^3}\right) + C$$

$$8*\int e^{\tan 2x} \tan 2x . \sec^2 2x \; dx$$

$$u = \tan 2x \quad , \quad dv = e^{\tan 2x} . \sec^2 2x \, dx$$

$$\frac{du}{dx} = 2 \sec^2 2x \quad , \quad \int dv = \frac{1}{2} \int e^{\tan 2x} . 2 \sec^2 2x \, dx$$

$$du = 2 \sec^2 2x \, dx \quad , \quad v = \frac{1}{2} e^{\tan 2x}$$

$$\int e^{\tan 2x} \tan 2x . \sec^2 2x \; dx = \tan 2x \cdot \frac{1}{2} e^{\tan 2x} - \frac{1}{2} \int e^{\tan 2x} . 2 \sec^2 2x \, dx$$

$$= \tan 2x \cdot \frac{1}{2} e^{\tan 2x} - \frac{1}{2} e^{\tan 2x} + C$$

$$9 * \int \frac{\ln x}{x^2}\, d\,x$$

$$u = \ln x \qquad , \quad d\,v = \frac{1}{x^2} d\,x$$

$$\frac{d\,u}{d\,x} = \frac{1}{x} \qquad , \quad \int \; d\,v = \int \frac{1}{x^2} d\,x$$

$$d\,u = \frac{1}{x} d\,x \qquad , \quad v = -\frac{1}{x}$$

$$\int \frac{\ln x}{x^2}\, d\,x = \ln x \quad (-\frac{1}{x}) \quad - \int -\frac{1}{x} \quad . \frac{1}{x} d\,x$$

$$= -\frac{1}{x} \ln x \quad + \int \frac{1}{x^2} d\,x$$

$$= -\frac{1}{x} \ln x \quad -\frac{1}{x} + C$$

$$10 * \int \sec^3 x\, d x = \int \sec^2 x \sec x\, d x = \int (1+\tan^2 x) \sec x\, d x$$

$$= \int (\sec x + \tan^2 x . \sec x) d x$$

$$= \int \sec x\, d x + \int \tan x (\tan x . \sec x) d x$$

$$\int \sec x\, d x = \int \sec x \times \frac{\sec x + \tan x}{\sec x + \tan x} d x = \int \frac{\sec^2 x + \sec x . \tan x}{\sec x + \tan x} d x$$

$$= \ln (\tan x + \sec x)$$

$$\int \tan x (\tan x . \sec x) d x$$

$$u = \tan x \qquad , \qquad d v = \tan x . \sec x\, d x$$

$$\frac{d u}{d x} = \sec^2 x \qquad , \qquad \int d v = \int \tan x . \sec x\, d x$$

$$d u = \sec^2 x\, d x \qquad , \qquad v = \sec x$$

$$\int \tan x (\tan x . \sec x) d x = \tan x . \sec x - \int \sec x . \sec^2 x\, d x$$

$$= \tan x . \sec x - \int \sec^3 x\, d x$$

$$\int \sec^3 x \, dx = \ln\left(\tan x + \sec x \right) + \tan x . \sec x - \int \sec^3 x \, dx$$

$$3 \int \sec^3 x \, dx = \ln\left(\tan x + \sec x \right) + \tan x . \sec x$$

$$\int \sec^3 x \, dx = \frac{1}{3} \left[\ln\left(\tan x + \sec x \right) + \tan x . \sec x \right] + C$$

$$11*\int \frac{x^{2}\tan^{-1}x^{3}}{x^{6}+1}\,dx = \frac{1}{3}\int (\tan^{-1}x^{3}).\,\frac{3x^{2}}{x^{6}+1}\,dx = \frac{1}{6}(\tan^{-1}x^{3})^{2}+C$$

$$12*\int \frac{x\,(\sin^{-1}x^{2})^{6}}{\sqrt{1-x^{4}}}\,dx = \frac{1}{2}\int (\sin^{-1}x^{2})^{6}.\,\frac{2x}{\sqrt{1-x^{6}}}\,dx = \frac{1}{14}(\sin^{-1}x^{2})^{7}+C$$

$$13*\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^{\sqrt{x}}\,\frac{1}{2}\,x^{-\frac{1}{2}}\,dx = 2\,e^{\sqrt{x}}+C$$

$$14*\int \frac{e^{\frac{1}{x}}}{x^{2}}\,dx = -\int e^{\frac{1}{x}}\,(-\tfrac{1}{x^{2}})\,dx = -e^{\frac{1}{x}}+C$$

$$15*\int \frac{(2-4\ln x)^{3}}{x}\,dx = -\frac{1}{4}\int (2-4\ln x)^{4}.(-4\frac{1}{x})\,dx = -\frac{1}{20}(2-4\ln x)^{5}+C$$

$$16*\int \frac{(\sqrt{x}-5)^{3}}{\sqrt{x}}\,dx = 2\int (\sqrt{x}-5)^{3}.(\frac{1}{2}x^{-\frac{1}{2}})\,dx = \frac{1}{2}(\sqrt{x}-5)^{4}+C$$

$$17*\int x^3 e^{1-2x^2}\,dx=\int x^2 e^{1-2x^2}\,x\,dx$$

$$u = x^2 \qquad , \quad dv = e^{1-2x^2}.x\,dx$$

$$\frac{du}{dx} = 2x \qquad , \quad \int dv = -\frac{1}{4}\int e^{1-2x^2}.(-4x)\,dx$$

$$du = 2x\,dx \qquad , \quad v = -\frac{1}{4}e^{1-2x^2}$$

$$\int x^3 e^{1-2x^2}\,dx = x^2\left(-\frac{1}{4}e^{1-2x^2}\right) - \int -\frac{1}{4}e^{1-2x^2}\,2x\,dx$$

$$= -\frac{1}{4}x^2 e^{1-2x^2} - \frac{1}{8}\int e^{1-2x^2}(-4x)\,dx$$

$$= -\frac{1}{4}x^2 e^{1-2x^2} - \frac{1}{8}e^{1-2x^2} + C$$

$$18 * \int \frac{x^5}{\sqrt{1-2x^3}}\,dx = \int x^3 (1-2x^3)^{-\frac{1}{2}}\; x^2\,dx$$

$$u = x^3 \quad , \quad dv = (1-2x^3)^{-\frac{1}{2}} . x^2\,dx$$

$$\frac{du}{dx} = 3x^2 \quad , \quad \int dv = -\frac{1}{6}\int (1-2x^3)^{-\frac{1}{2}} .(-6x^2)\,dx$$

$$du = 3x^2\,dx \quad , \quad v = -\frac{1}{3}(1-2x^2)^{\frac{1}{2}}$$

$$\int \frac{x^5}{\sqrt{1-2x^3}}\,dx = x^3 \left(-\frac{1}{3}(1-2x^2)^{\frac{1}{2}} \right) - \int -\frac{1}{3}(1-2x^3)^{\frac{1}{2}} .(3x^2)\,dx$$

$$= -\frac{1}{12}x^3(1-2x^2)^{\frac{1}{2}} - \frac{1}{6}\int (1-2x^3)^{\frac{1}{2}} .(-6x^2)\,dx$$

$$= -\frac{1}{12}x^3(1-2x^2)^{\frac{1}{2}} - \frac{1}{9}(1-2x^2)^{\frac{3}{2}} + C$$

$$19 * \int x^{3} \sin(1-x^{2})\,dx \quad = \int x^{2} \sin(1-x^{2})\,x\,dx$$

$$u = x^{3} \quad , \quad dv = \sin(1-x^{2}).x\,dx$$

$$\frac{du}{dx} = 3x^{2} \quad , \quad \int dv = -\frac{1}{2}\int \sin(1-x^{2}).(-x^{2})\,dx$$

$$du = 3x^{2}\,dx \quad , \quad v = -\frac{1}{2}(-\cos(1-x^{2})) = \frac{1}{2}\cos(1-x^{2})$$

$$\int x^{3}\sin(1-x^{2})\,dx = x^{3} \,.\, \frac{1}{2}\cos(1-x^{2}) - \int \frac{1}{2}\cos(1-x^{2})\ 3x^{2}\,dx$$

$$= \frac{1}{2}x^{3}\cos(1-x^{2}) + \frac{3}{4}\int \cos(1-x^{2})(-2x)\,dx$$

$$= \frac{1}{2}x^{3}\cos(1-x^{2}) + \frac{3}{4}\sin(1-x^{2}) + C$$