WELLCOME

TO

FOUNDATION YEAR (11 / 2023)
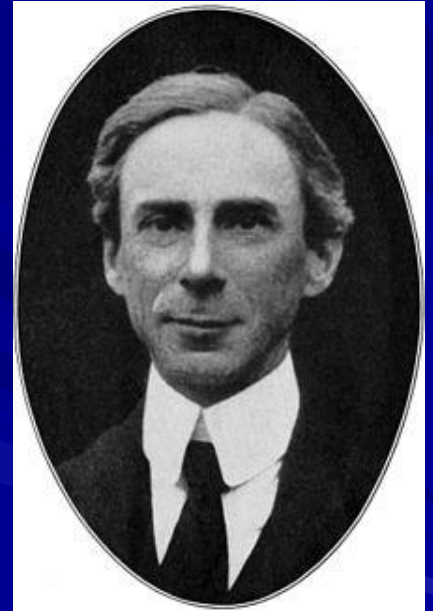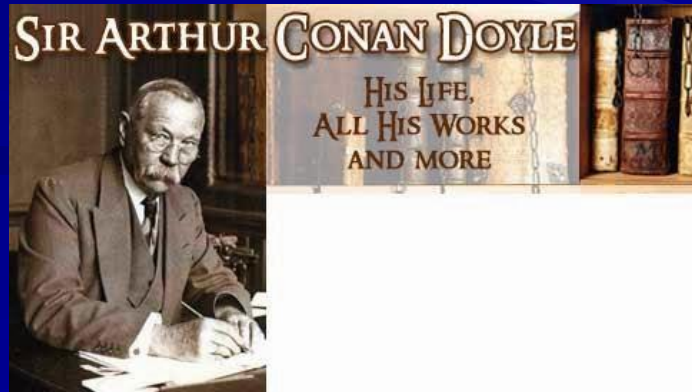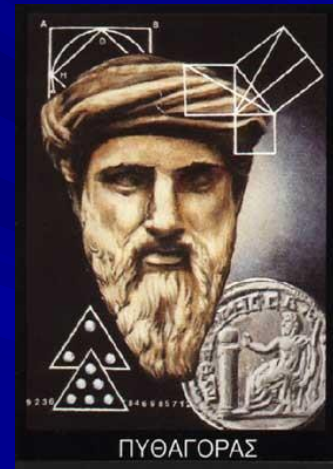
Mathematics

From the drop of water, a logician could infer the possibility of an Atalantic Ocean or Nigara Fall without having seen or heard of the one or the other.

Arthur Conan Doyle : sherlock Holmes

SIR ARTHUR CONAN DOYLE
HIS LIFE, ALL HIS WORKS AND MORE

Number is the guide and master of human thought. Without its power, everything would remain obsure and confused

Pythagora

*Mathematical Logic   Individual Assessment &

                                   Group Assessment

*Probability   Individual Assessment &

Permutation & combination   Group Assessment

Conditional Probability

*Statistics   Individual Assessment , Group Assessment
                                   & Individual Assignment

*Discrete Probability   Individual Assessment , Group Assessment
                                   & Individual Assignment

Discrete , Binomial , Poisson

*Continuous Probability   Individual Assessment &

                                   Group Assessment

Continuous , Normal

- Continuous Assessment Individual Assessment (5 times) 30 % , Group

  Assessment (5 times)10% , Individual Assessment (2 times) 10% &
End semester theory Assessment 50%

Mathematical Logic

Statement

1 . Statements may be sentences (or) equations (or)

inequations (or) identities.

2 . A statement may be true or false , but not both

at the same time.

3 . The opposite of false is true and opposite of

true is false.

Logical Connectives

$\wedge$ *and* , *Conjunction*

$\vee$ *or* , *Disjunction*

$\Rightarrow$ *implies* , *Conditional*

$\Leftrightarrow$ *if and only if* , *Biconditional*

$: ,$ $\mid$ *such that*

$\sim ,$ *not , opposite , negation*

$\equiv ,$ *Equivalence*

Truth Table

P	q	$P \wedge q$	$P \vee q$	$P \Rightarrow q$	$P \Leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Truth Table

P	q	$\sim p$	$\sim q$	$\sim (p \wedge q)$	$\sim (P \Leftrightarrow q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

Let p be "He is tall" and q be "He is handsome". Write each of the following statements in symbolic form using p and q and find its truth value by using table.

(i) He is tall but not handsome.

(ii) It is false that he is short or handsome

(iii) He is neither tall nor handsome

(iv) He is tall, or he is short and handsome

(v) It is trute that he is short or not handsome

$(i)\, p \wedge \sim q$ $(ii)\, \sim (\sim p \vee q)$ $(iii)\, \sim p \wedge \sim q\, (or)\, \sim (p \vee q)$

$(iv)\, p \vee (\sim p \wedge q)$ $(v)\, \sim p \vee \sim q\, (or)\, \sim (p \wedge q)$

$(i)\ p\ \wedge \sim q$

P	q	$\sim q$	$p\ \wedge\ \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

$(ii) \sim (\sim p \lor q)$

P	q	$\sim P$	$\sim p \lor q$	$\sim (\sim p \lor q)$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

$(iii) \sim p \wedge \sim q$

P	q	$\sim P$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(iv) $p \vee (\sim p \wedge q)$

P	q	$\sim P$	$(\sim p \wedge q)$	$p \vee (\sim p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

$(v) \sim p \lor \sim q$

P	q	$\sim P$	$\sim q$	$\sim p \lor \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Determine the truth value of each of the following composite statements.

(i) If $3 + 2 = 7$, then $4+4 =8$

p : $3 + 2 = 7$,   q : $4+4 =8$

$p \Rightarrow q$

p	q	$p \Rightarrow q$
F	T	T

(ii) It is not true that $2 + 2 = 5$, if and only if $4+4 =10$

p : $2 + 2 = 5$,   q : $4+4 =10$

$\sim (p \Leftrightarrow q)$

p	q	$p \Leftrightarrow q$	$\sim (p \Leftrightarrow q)$
F	F	T	F

(iii) Yangon is in India or Tokyo is in Myanmar

p : Yangon is in India ,   q : Tokyo is in Myanmar

$(p \vee q)$

p	q	$p \vee q$
F	F	F

(iv) It is not true that $1 + 1 = 3$ or $2 + 1 = 3$

p : $1 + 1 = 3$,   q : $2+1=3$

$\sim (p \vee q)$

p	q	$p \vee q$	$\sim (p \vee q)$
F	T	T	F

(v) It is false that   Yangon is in Japan then Paris is in Myanmar

p : Yangon is in Japan ,   q : Paris is in Myanmar

$\sim (p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\sim (p \Rightarrow q)$
F	F	T	F

Propostion and Truth Table

$$(\, p \wedge q \,) \Rightarrow (\, p \vee q \,)$$

P	q	$p \wedge q$	$p \vee q$	$(\, p \wedge q \,) \Rightarrow (\, p \vee q \,)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

(iv) $(\sim p \lor q) \land \sim r$

P	q	r	$\sim p$	$(\sim p \lor q)$	$\sim r$	$(\sim p \lor q) \land \sim r$
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

$(p \Rightarrow q) \vee \sim (p \Leftrightarrow \sim q)$

P	q	$p \Rightarrow q$	$\sim q$	$(p \Leftrightarrow \sim q)$	$\sim (p \Leftrightarrow \sim q)$	$(p \Rightarrow q) \vee \sim (p \Leftrightarrow \sim q)$
T	T	T	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	T
F	F	T	T	F	T	T

Logical Equivalence

　　Two propositions P (p , q , r ,...) and Q (p , q , r ,....) are said to be logically equivalent if their truth tables are indentical. We denote this by $P \equiv Q$

Verify that $p \Rightarrow \sim q \equiv q \Rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \Rightarrow \sim q$	$q \Rightarrow \sim p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T
				*	*

*By the * columns* $p \Rightarrow \sim q \equiv q \Rightarrow \sim p$

Verify : $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

* *

By the * columns, they are equivalent

Verify : $\sim (p \Leftrightarrow q) \equiv (p \Leftrightarrow \sim q) \equiv (q \Leftrightarrow \sim p)$

p	q	$\sim p$	$\sim q$	$p \Leftrightarrow q$	$\sim (p \Leftrightarrow q)$	$p \Leftrightarrow \sim q$	$q \Leftrightarrow \sim p$
T	T	F	F	T	F	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	T	F	F	F
					*	*	*

By the * columns, they are equivalent

Validity of Argument

A statement that a set of propositions $P_1, P_2, P_3, \ldots, P_n$ yields another proposition P is called an argument on propositions.

The propositions $P_1, P_2, P_3, \ldots, P_n$ are called premises and the proposition P is called concusion.

Argument form is shown as follows;

$$P_1 (p, q, r, \ldots)$$
$$P_2 (p, q, r, \ldots)$$
$$.$$
$$.$$
$$.$$
$$P_n (p, q, r, \ldots)$$

$\left. \right\}$ *premises*

$$\overline{}$$
$$P (p, q, r, \ldots)$$

Conclusion

Since an argument is also a statement it has a true value.

An argument on propositions is said to be valid if the conclusion P is the true whenver the premises $P_1, P_2, P_3, ..., P_n$ are true

Example:

If everyone does not have love, he is lonely. A person who fells loneliness can early die. Therefore, if he has no love, he can early die.

Let p be "A person who does not have love"

q be "A person who is lonly"

r be "A person who can early die"

$$p \Rightarrow q$$
$$q \Rightarrow r$$
$$\left.\begin{array}{c} \end{array}\right\} \; premises$$

$$\therefore p \Rightarrow r \qquad Conclusion$$

			P_1	P_2	P	
p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	
T	T	T	T	T	T	
T	T	F	T	F	F	
T	F	T	F	T	T	
T	F	F	F	T	F	
F	T	T	T	T	T	
F	T	F	T	F	T	
F	F	T	T	T	T	
F	F	F	T	T	T	

In 1st ,5th , 7th and 8th rows, premises are true then conclusions are true. Therefore, this argument is valid.

Test the validity of the argument. If it rain, then Mg Mya will be sick. It did not rain. Therefore Mg Mya was not sick.

Solution:

Let p be " It rain "

q be " Mg Mya is sick "

$$p \Rightarrow q$$
$$\sim p$$

} *premises*

$$\overline{}$$
$$\sim q \qquad Conclusion$$

p	q	$p \Rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

*

In 4[h] row, premises is true then conclusions is false. Therefore, this argument is not valid.

$$p \Rightarrow q$$

$$r \Rightarrow \sim q$$

$$\left.\begin{array}{c} \\ \end{array}\right\} \quad premises$$

$$\therefore r \Rightarrow \sim p \qquad Conclusion$$

p	q	r	$\sim p$	$\sim q$	P_1 $p \Rightarrow q$	P_2 $r \Rightarrow \sim q$	P $r \Rightarrow \sim p$	
T	T	T	F	F	T	F	F	
T	T	F	F	F	T	T	T	*
T	F	T	F	T	F	T	T	
T	F	F	F	T	F	T	T	
F	T	T	T	F	T	F	T	
F	T	F	T	F	T	T	T	*
F	F	T	T	T	T	T	T	*
F	F	F	T	T	T	T	T	*

By the * rows, the argument is valid

If Ma Aye gets married, then either Ma Hla is maid of honour or Ma Mya maid of honour. If Ma Hla is maid of honour and Ma Mya is maid of honour, then there will be a quarrel at the wedding. Therefore if Ma Aye gets married, then there will be quarrel at the wedding.

p : Ma Aye gets married

q : Ma Hla is maid of honour

r : Ma Mya maid of honour

s : there is a quarrel at the wedding

$$\left. \begin{array}{l} p \Rightarrow q \vee r \\ q \wedge r \Rightarrow s \end{array} \right\} \text{premises}$$

$$\overline{\therefore \ p \Rightarrow s} \qquad Conclusion$$

p	q	r	s	$q \lor r$	$q \land r$	$p \Rightarrow (q \lor r)$	$(q \land r) \Rightarrow s$	$p \Rightarrow s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	F
T	T	F	T	T	F	T	T	T
T	T	F	F	T	F	T	T	F
T	F	T	T	T	F	T	T	T
T	F	T	F	T	F	T	T	F
T	F	F	T	F	F	F	T	T
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	F	T
F	T	F	T	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	T	F	T	F	T	T	T
F	F	F	T	F	F	T	T	T
F	F	F	F	F	F	T	T	T

*

In 4^h row, premises is true then conclusions is false. Therefore, this argument is not valid.

In 4^h and 6th rows, premises are true then conclusions are false. Therefore, this argument is not valid.

Quantifiers

Universal Quantifier

$$(\forall x \in U) p(x) \quad or \quad \forall x, p(x)$$

$$If \ \{x / \ x \in U) p(x) \} = U \ , then \ \forall x, p(x) \ is \ true$$

$$If \ \{x / \ x \in U) p(x) \} \neq U \ , then \ \forall x, p(x) \ is \ flase$$

$$(\exists\, x \in U\,)\, p\,(\,x\,) \quad or \quad \exists\, x, p\,(\,x\,)$$

$$If\ \{x/\ \ x \in U\,)\, p\,(\,x\,)\,\} \neq \phi\,,\ then\ \exists\, x, p\,(\,x\,)\ is\ true$$

$$If\ \{x/\ \ x \in U\,)\, p\,(\,x\,)\,\} = \phi\,,\ then\ \exists\, x, p\,(\,x\,)\ is\ flase$$

Negation of Proposition Which contain quantifiers

Let $p\,(\,x\,)$ be a statement. The equivalent form of a negation of a statement which contain quantifiers is given by the following theorem.

$$(1)\ \sim (\,\forall\, x \in U\,)\, p\,(\,x\,)\,\} \equiv (\,\exists\, x \in U\,) \sim p\,(\,x\,)$$

$$(2)\ \sim (\exists\, x \in U\,)\, p\,(\,x\,)\ \equiv (\,\forall\, x \in U\,) \sim p\,(\,x\,)$$

Determine the truth value of each of the following. Here the universal set $U = R$

$(\,i\,) \quad \forall\ x\,, \left|\ x\ \right| =\ x$

$\quad\quad Since \quad , \left|-\ 1\ \right| \neq\ -1$

$\quad\quad T_{\,p}\ \neq\ R$

$\quad\quad$ Therefore, it is false

$(\,ii\,) \quad \exists\ x\,, x^{\,2} =\ x$

$\quad\quad Since \quad , 1^{\,2}\ =\ 1$

$\quad\quad T_{\,p}\ \neq\ \phi$

$\quad\quad$ Therefore , it is true

(iii) $\forall$ x , x $+1>$ x

$T_p = R$

Therefore, it is true

(iv) $\exists$ x , $|$ x $|= 0$

$Since$, $|0| = 0$

$T_p \neq \phi$

Therefore, it is true

(v) $\exists$ x , x $+$ 2 $=$ x

$T_p = \phi$

Therefore , it is false

(vi) $\forall$ x , x $+$ 2 > 5

$Since$, 1 $+$ 2 $<$ 5

$T_p \neq R$

Therefore, it is false

Prove that the following statements are false. Here $U = R =$ the set of real numbers.

(i) $\forall x$, $x^2 - 3x + 2 = 0$

$\quad T_p = \{1, 2\} \neq R$

Therefore, it is false

(ii) $\forall x$, $5x + 1 = 2x + 9$

$\quad T_p = \{\dfrac{8}{3}\} \neq R$

Therefore, it is false

(iii) $\forall x$, $x^2 + 9 = (x + 3)(x - 3)$

$\quad T_p = \phi \neq R$

Therefore, it is false

(iv) $\exists x$, $x^2 - 4 \neq (x + 2)(x - 2)$

$\quad T_p = \phi$

Therefore, it is false

(v) $\exists x$, $x \neq x$

$\quad T_p = \phi$

Therefore, it is false

Find the counter example for each of the following statement where A = { 2 , 3 , 4 , 5 , 6 , 7 ,8 , 9 }

(i) $\forall x \in A$, $x + 5 < 12$

Take $9 \in A$,

then $9 + 5 > 12$ is false

$T_p \neq A$

Therefore , this statement is false

(ii) $\forall x \in A$, x is prime

Take $9 \in A$,

then   9 is not prime

$T_p \neq A$

Therefore , this statement is false

(iii) $\forall x \in A$, x is even

Take $9 \in A$,

then   9 is not even

$T_p \neq A$

Therefore , this statement is false

(iv) $\forall x \in A$, $|x - 4| < 3$

Take $9 \in A$,

then $|9 - 4| > 3$

$T_p \neq A$

Therefore , this statement is false

(v) $\forall x \in A$, $x^2 - 1 > 7$

Take $2 \in A$,

then $2^2 - 4 < 7$

$T_p \neq A$

Therefore , this statement is false

Negate each of the statement .

(i) $\forall x, |x| = x$

$\sim (\forall x, |x| = x) \equiv \exists x, |x| \neq x$

(ii) $\exists x, x^2 = x$

$\sim (\exists x, x^2 = x) \equiv \forall x, x^2 \neq x$

(iii) $\forall x, x+1 > x$

$\sim (\forall x, x+1 > x) \equiv \exists x, x+1 \leq x$

(iv) $\exists x, |x| = 0$

$\sim (\exists x, |x| = 0) \equiv \forall x, |x| \neq 0$

$(v) \ \exists \, x \, , \, x \, + \, 2 \, = \, x$

$\sim (\exists \, x \, , \, x \, + \, 2 \, = \, x \,) \, \equiv \quad \forall \, x \, , \, x \, + \, 2 \, \neq \, x$

$(vi) \ \forall \, x \, , \, x \, + \, 2 \, > \, 5$

$\sim (\forall \, x \, , \, x \, + \, 2 \, > \, 5 \,) \, \equiv \quad \exists \, x \, , \, x \, + \, 2 \, \leq \, 5$

State the Pythagoras Theorem with logical notations

$$\forall \, \Delta ABC : \hat{C} = 90^{\circ} \quad \Leftrightarrow \quad AB^2 = AC^2 + BC^2$$

Re-write the statement " a divides b if and only if some integer n, b = na " by using logic notations.

$$a \ divides \ b \ \Leftrightarrow \exists \, n \in J : b = n\,a$$

$$In \, \Delta ABC \, , \hat{A} = 30^{\circ} \, , \hat{B} = 60^{\circ} \quad \Rightarrow \quad \hat{C} = 90^{\circ}$$

$$\exists \, \Delta ABC : \hat{A} = 30^{\circ} \, , \hat{B} = 60^{\circ} \Leftrightarrow \hat{C} = 90^{\circ}$$

$$\underline{\exists} \, (x,y) : x + y = 10$$

$$\underline{\forall} \, (x,y) : x > y \Leftrightarrow y < x$$

(i) Fill the blank $\sim \left(\forall x , |x| = x \right) \equiv$ _____

$\sim \left(\forall x , |x| = x \right) \equiv \exists x , |x| \neq x$

(ii) Fill the blank $\sim \left(\exists \triangle ABC , AB + BC \leq AC \right) \equiv$ _____

$\sim \left(\exists \triangle ABC , AB + BC \leq AC \right) \equiv \forall \triangle ABC , AB + BC < AC$

(iii) Fill the blank $\Rightarrow$ or $\Leftrightarrow$: $x = 0$ and $y = 0$ ____ $x\,y = 0$

$x = 0$ and $y = 0$ $\Rightarrow$ $x\,y = 0$

(iv) Fill the blank $\forall$ or $\exists$: Any surface

___ $\triangle ABC : \angle A + \angle B + \angle C = 180$

$\exists \triangle ABC : \angle A + \angle B + \angle C = 180$

(v) Fill the blank $\Rightarrow$ or $\Leftrightarrow$: $y = x^n$ ____ $\dfrac{d\,y}{d\,x} = n\,x^{n-1}$

$y = x^n$ $\Rightarrow$ $\dfrac{d\,y}{d\,x} = n\,x^{n-1}$

* Over the universe of four-wheelers, let p (x) : x is a four wheeler. p (x) : x is a car and r (x) : x is a manufactured by Maruti Udyog L.td. Express the following statements using quantifires

(a) Every car is a four-wheeler manufactured by MUL.

$$\forall x : (p (x) \wedge q (x)) \Rightarrow r (x)$$

(b) There are cars that are not manufactured by MUL.

$$\exists x : p (x) \wedge \sim r (x)$$

(c) Every four-wheeler is a car.

$$\forall x : p (x) \Rightarrow r (x)$$

* Over the universe of the animals, let p (x) : x is a whale, q (x) : x is fish and r (x) : x lives in water.

(a) There exists an animal which does not live in water.

$$\exists x : \sim r(x)$$

(b) There exists a fish that is not a whale.

$$\exists x : q(x) \wedge \sim p(x)$$

(c) Every whale that lives in the water is fish.

$$\forall x : p(x) \wedge r(x) \Rightarrow q(x)$$