Discrete Probability Distribution

If a variable X can assume a discrete set of values $x_1, x_2, \ldots\ldots, x_K$ with respective probabilities $p_1, p_2, \ldots, p_k$ where

$$p_1 + p_2 + \ldots\ldots + p_K = 1,$$

we say that a discrete probability distribution for X.

Consider, Two dice are thrown

(1 , 1)	(1 , 2)	(1 , 3)	(1 , 4)	(1 , 5)	(1 , 6)
(2 , 1)	(2 , 2)	(2 , 3)	(2 , 4)	(2 , 5)	(2 , 6)
(3 , 1)	(3 , 2)	(3 , 3)	(3 , 4)	(3 , 5)	(3 , 6)
(4 , 1)	(4 , 2)	(4 , 3)	(4 , 4)	(4 , 5)	(4 , 6)
(5 , 1)	(5 , 2)	(5 , 3)	(5 , 4)	(5 , 5)	(5 , 6)
(6 , 1)	(6 , 2)	(6 , 3)	(6 , 4)	(6 , 5)	(6 , 6)

Let X be the sum of score on the dice

$$P_k = P(x = k)$$

$$P_2 = \frac{1}{36}, \quad P_3 = \frac{2}{36}, \quad P_4 = \frac{3}{36}, \quad P_5 = \frac{4}{36}, \quad P_6 = \frac{5}{36}$$

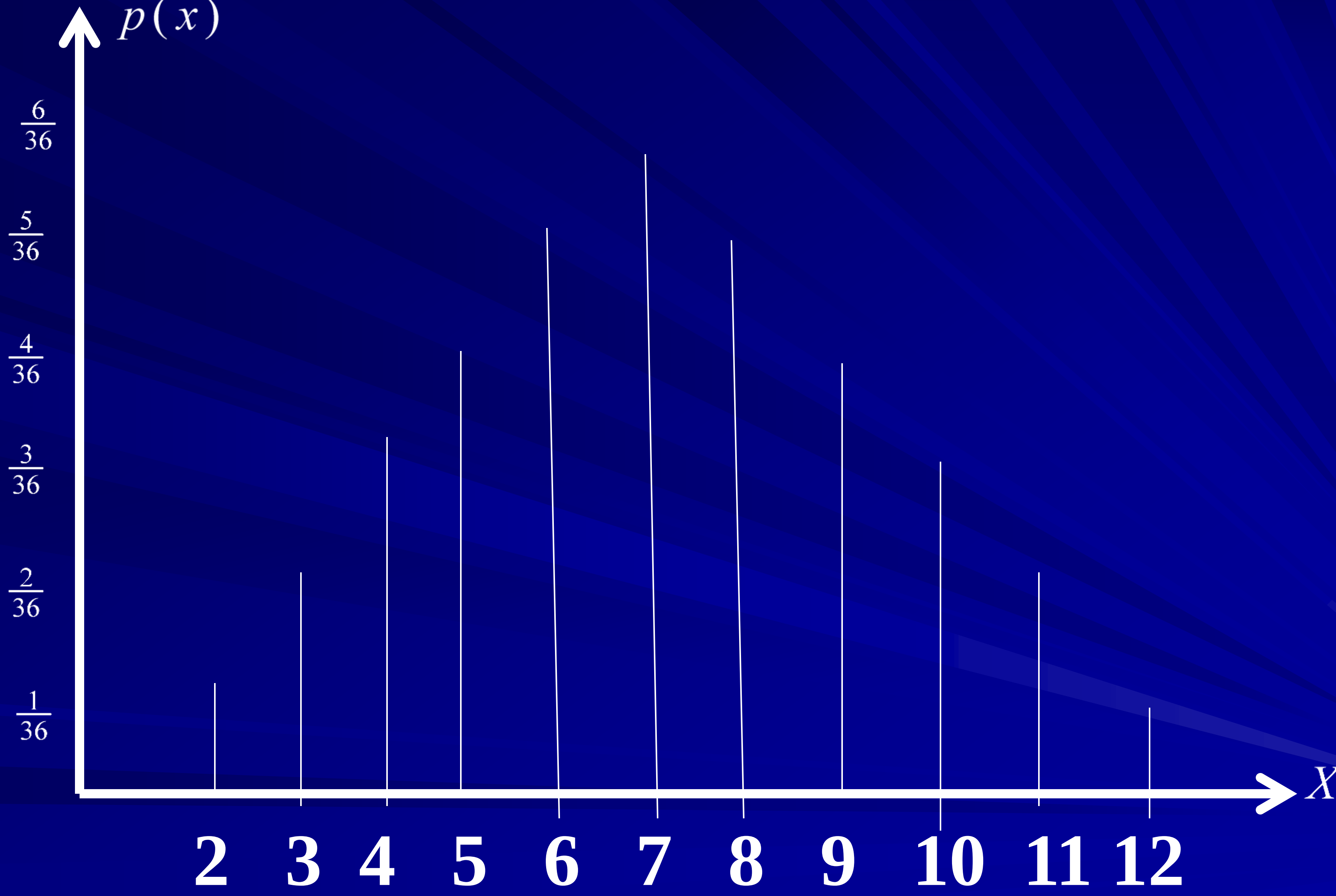
$$P_7 = \frac{6}{36}$$

$$P_8 = \frac{5}{36}, \quad P_9 = \frac{4}{36}, \quad P_{10} = \frac{3}{36}, \quad P_{11} = \frac{2}{36}, \quad P_{12} = \frac{1}{36}$$

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$p(x) = \begin{cases} \dfrac{j-1}{36} & , \ j = 2,3,4,5,6,7 \\ \dfrac{13-j}{36} & , \ j=8,9,10,11,12 \\ 0 & , \text{ otherwise} \end{cases}$$

Graph



Mathematical Expectation

(mean , average)

$$\mu = E(X) = p_1 . x_1 + \ldots\ldots + p_k . x_k$$

$$E(X) = \sum_{j=1}^{k} p_j . x_j$$

$$E(3X+5) = \sum_{j=1}^{k} p_j .(3x_j + 5)$$

$$\mu = E(X^2) = p_1 . x_1^{\,2} + \ldots\ldots + p_k . x_k^{\,2}$$

$$E(X^2) = \sum_{j=1}^{k} p_j . x_j^{\,2}$$

Expected number of the sum of score

$$E(X) = \frac{1}{36}\times 2 + \frac{2}{36}\times 3 + \ldots\ldots + \frac{1}{36}\times 12$$

$$E(X^2) = \frac{1}{36}\times 2^2 + \frac{2}{36}\times 3^2 + \ldots\ldots + \frac{1}{36}\times 12^2$$

$$E(X^2) = \frac{1}{36}\times 2^2 + \frac{2}{36}\times 3^2 + \ldots\ldots + \frac{1}{36}\times 12^2$$

Some Results of E (X)

a and b are constant.

* $E(a) = a$

* $E(aX) = aE(X)$

* $E(aX + b) = aE(X) + b$

* $E\left[f(x) \pm g(x)\right] = E\left[f(x)\right] \pm E\left[g(x)\right]$

$e.g\ ,\ E(3X + 5) = 3E(X) + 5$

Expected number of getting 7 in 300 times .

Expected number of getting 7 in 300 times

$$= \frac{6}{36} \times 300 \quad = 50$$

Variance

$$Var(X) = E\left(X - \overline{X}\right)^2$$

$$Var(X) = E(x^2) - \{E(x)\}^2$$

$$Note\_\ E(x^2) \geq \{E(x)\}^2$$

Standard deviation

$$\sigma = \sqrt{Var(x)} \qquad Var(x) = \sigma^2$$

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

Some Results of Var (X)

a and b are constant.

* $Var(a) = 0$

* $Var(aX) = a^2 Var(X)$

* $Var(aX + b) = a^2 Var(X)$

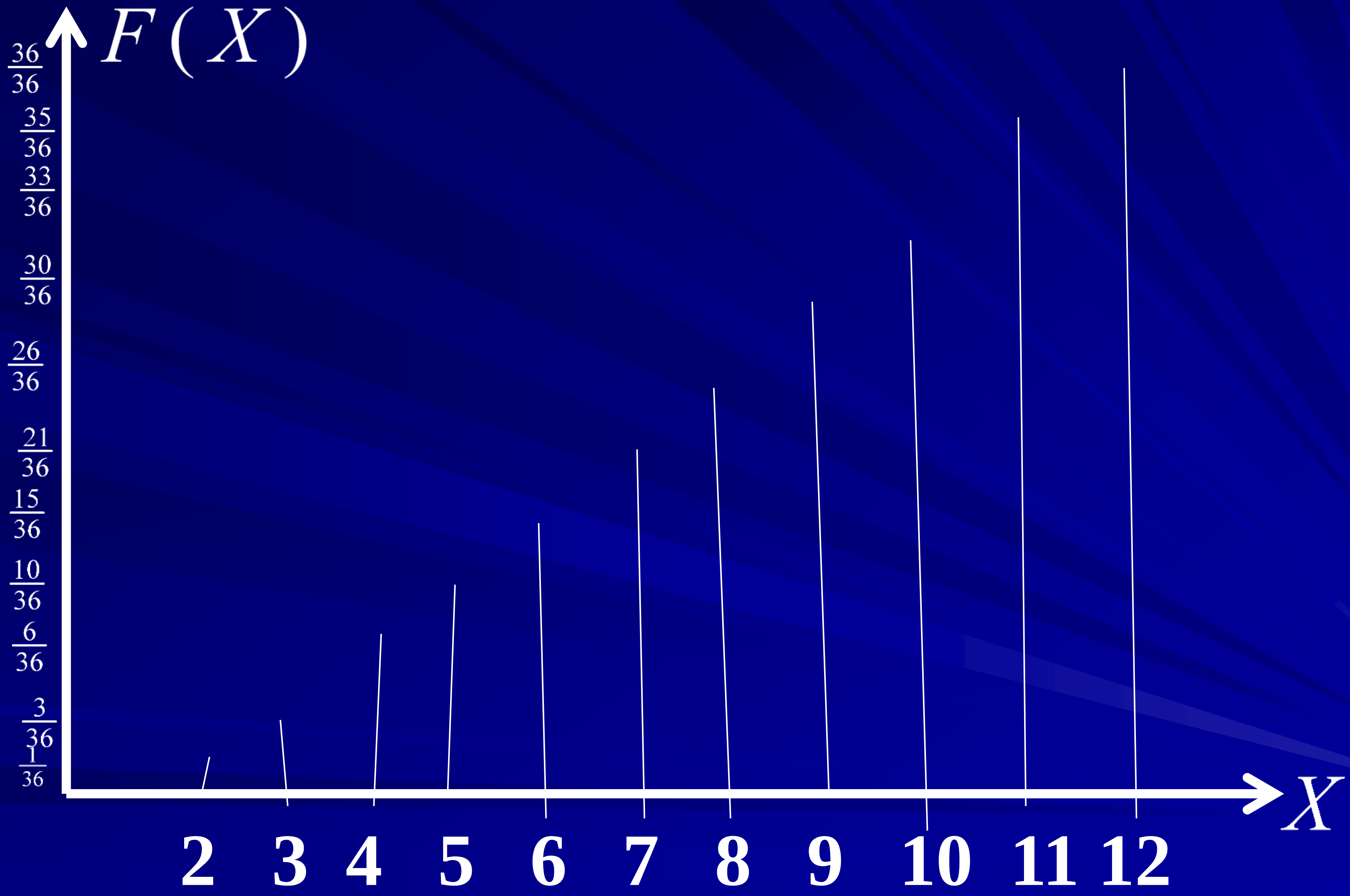
* $Var\left[f(x) \pm g(x)\right] = Var\left[f(x)\right] \pm Var\left[g(x)\right]$

$e.g\ ,\ Var(3X + 5) = 9\,Var(X)$

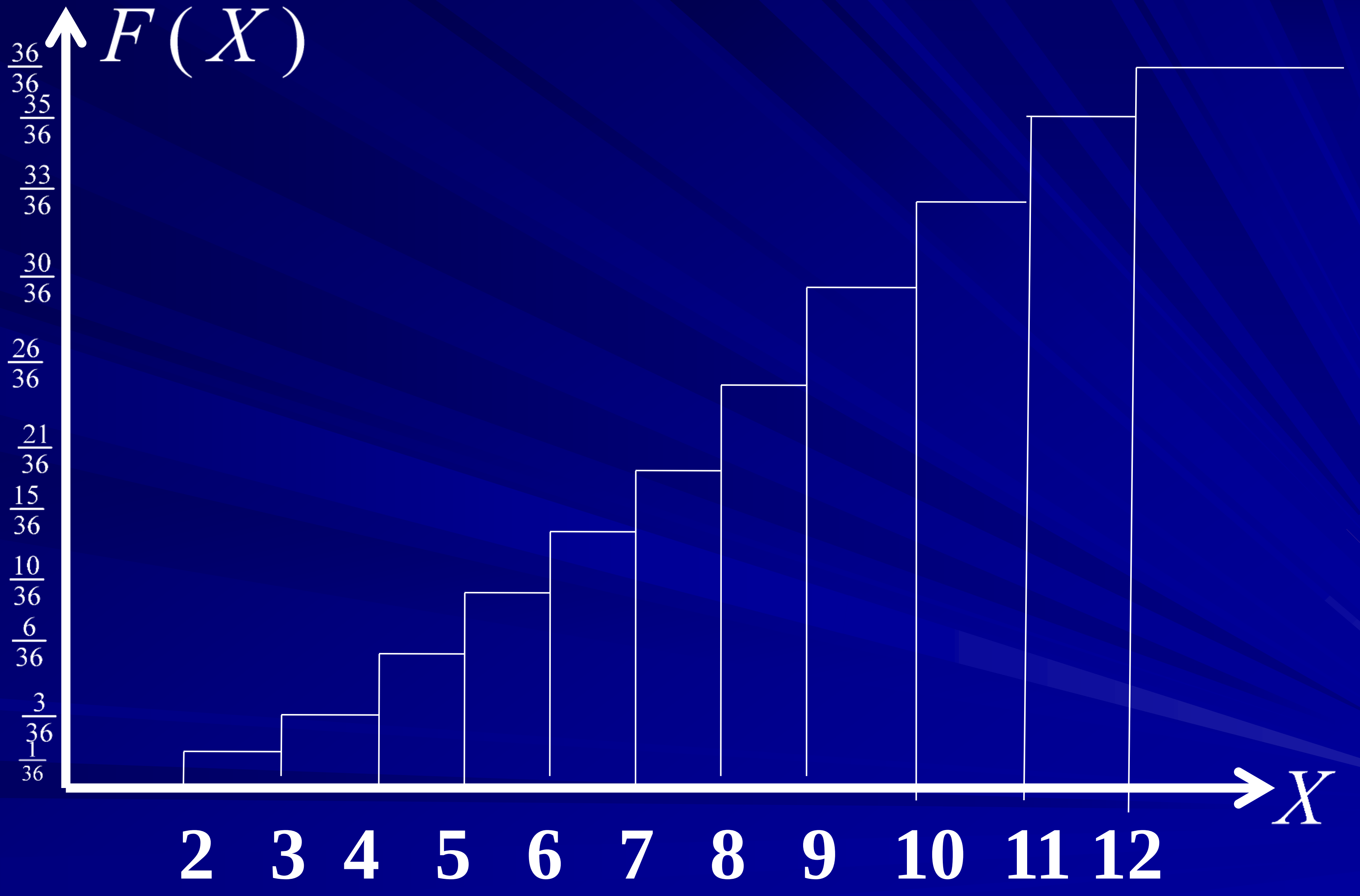
Cumulative Probability, F (X)

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
F (X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	$\frac{36}{36}$

Cumulative Graph



Cumulative Graph



Median

The median splits the area under the curve $y = f(x)$ into two halves. So if the value of the median is m ,

$$P(a \le x \le m) = \frac{1}{2} = 0.5$$

$$\text{i.e,} \quad F(m) = \frac{1}{2}$$

Pg 75 . EX (4.1) No. 1

If X is the random variable showing the number of boy in families with three children construct a table showing the probability distribution of X.

Pg 75 . EX (4.1) No. 1

X be the numbers of boys in family with three children

$$R_x = \{0,1,2,3\} \qquad P(b) = \frac{1}{2} \qquad \mathrm{P}(\mathrm{g}) = \frac{1}{2}$$

$$P(g,g,g) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(g,g,b) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(g,b,g) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(g,b,b) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(b,g,g) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(b,g,b) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(b,b,g) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(b,b,b) = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

$$P(x=0)=\frac{1}{8}$$

$$P(x=1)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(x=2)=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{3}{8}$$

$$P(x=3)=\frac{1}{8}$$

$$P_0 = P(x=0) = {}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

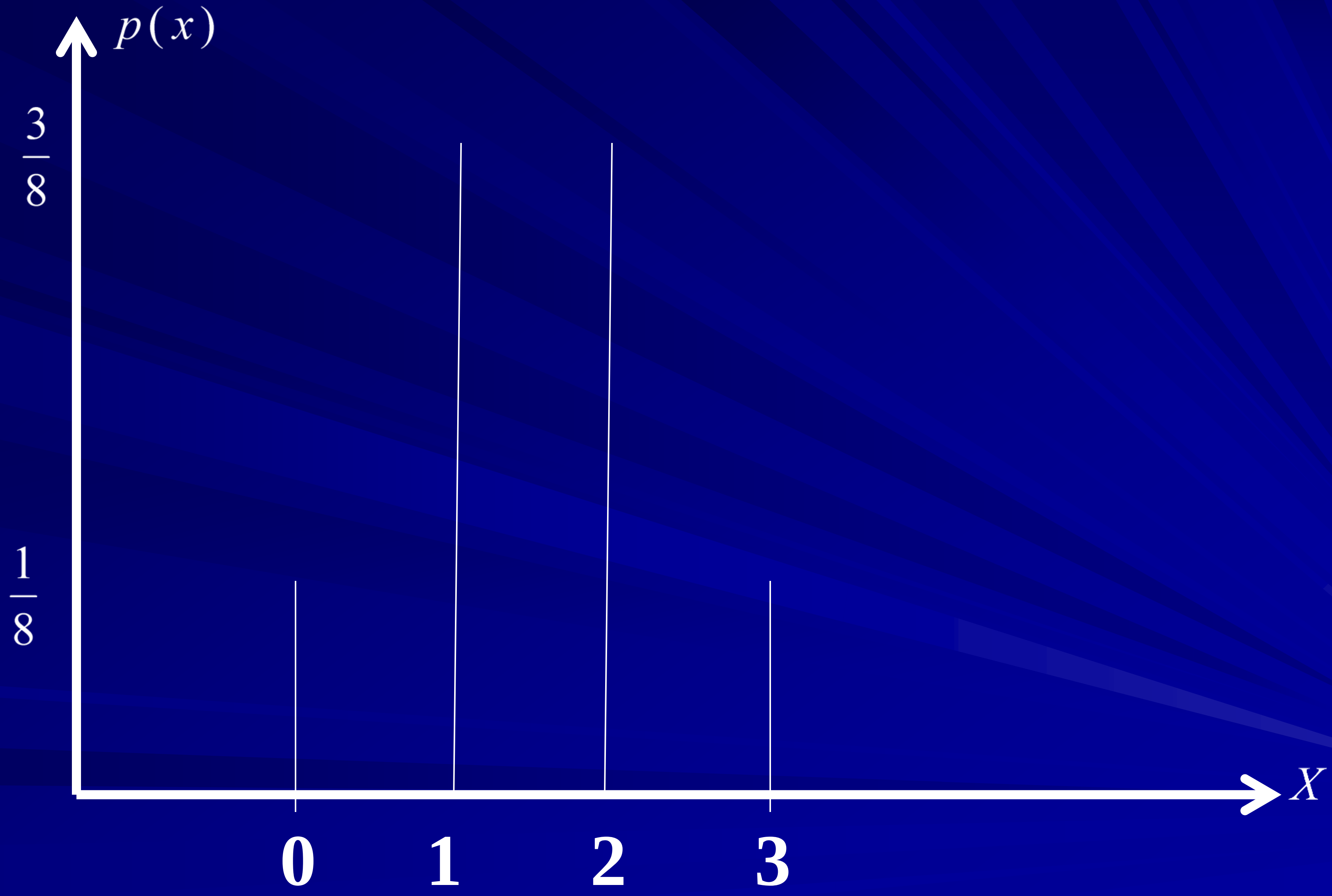
$$P_1 = P(x=1) = {}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P_2 = P(x=2) = {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P_3 = P(x=3) = {}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
F (X)	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{8}{8}$

Probability Graph



The expected number of boy

$$E(X) = \frac{1}{8}\times 0 + \frac{3}{8}\times 1 + \frac{3}{8}\times 2 + \frac{1}{8}\times 3 = \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = \frac{1}{8}\times 0^2 + \frac{3}{8}\times 1^2 + \frac{3}{8}\times 2^2 + \frac{1}{8}\times 3^2 = \frac{24}{8} = 3$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= 3 - \left[\tfrac{3}{2}\right]^2 = 3 - \frac{9}{4} = \frac{3}{4}$$

$$Standard\ deviation, \sigma = \sqrt{Var(X)} = \frac{\sqrt{3}}{2}$$

Pg 76 . EX (4.1) No. 3

Three marbles are draw without replacement from an urn containing 4 red and 6 white marbles. If X is a random variable which denotes the total number of red marbles drawn,

X be the number of red

$$R_x = \{0,1,2,3\} \qquad W = White\ marble \qquad R = \text{Re}d\ marble$$

$$P(W,W,W) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8} = \frac{5}{30} \qquad P(R,W,W) = \frac{4}{10}\times\frac{6}{9}\times\frac{5}{8} = \frac{5}{30}$$

$$P(W,W,R) = \frac{6}{10}\times\frac{5}{9}\times\frac{4}{8} = \frac{5}{30} \qquad P(R,W,R) = \frac{4}{10}\times\frac{6}{9}\times\frac{3}{8} = \frac{3}{30}$$

$$P(W,R,W) = \frac{6}{10}\times\frac{4}{9}\times\frac{5}{8} = \frac{5}{30} \qquad P(R,R,W) = \frac{4}{10}\times\frac{3}{9}\times\frac{6}{8} = \frac{3}{30}$$

$$P(W,R,R) = \frac{6}{10}\times\frac{4}{9}\times\frac{3}{8} = \frac{3}{30} \qquad P(R,R,R) = \frac{4}{10}\times\frac{3}{9}\times\frac{2}{8} = \frac{1}{30}$$

$$P_0 = P(x=0) = \frac{5}{30}$$

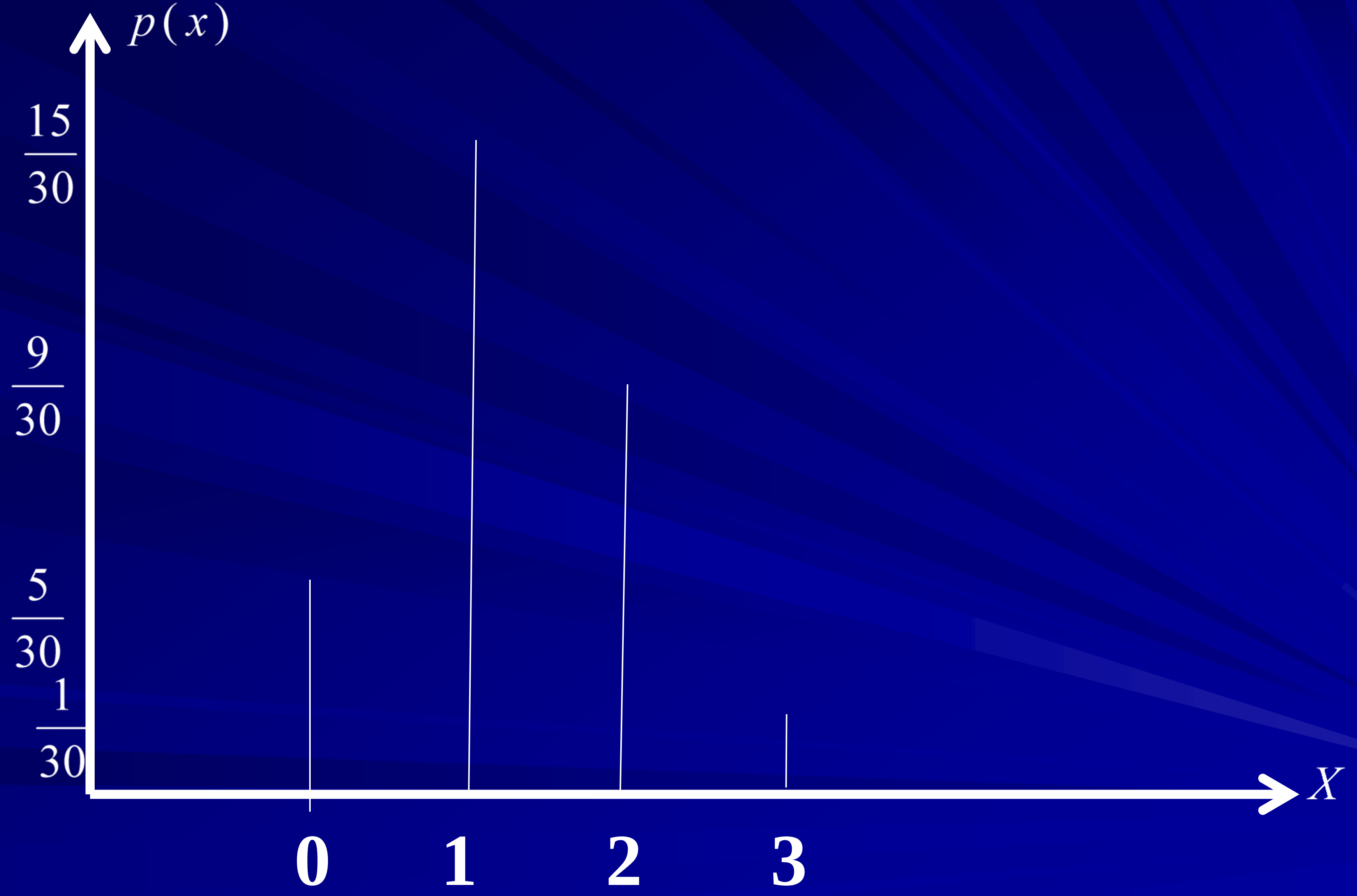
$$P_1 = P(x=1) = \frac{5}{30} + \frac{5}{30} + \frac{5}{30} = \frac{15}{30}$$

$$P_2 = P(x=2) = \frac{3}{30} + \frac{3}{30} + \frac{3}{30} = \frac{9}{30}$$

$$P_3 = P(x=3) = \frac{1}{30}$$

X	0	1	2	3
P(X)	$\frac{5}{30}$	$\frac{15}{30}$	$\frac{9}{30}$	$\frac{1}{30}$
F (X)	$\frac{5}{30}$	$\frac{20}{30}$	$\frac{29}{30}$	$\frac{30}{30}$

Probability Graph



The expected number of red marbles

$$E(X) = \frac{5}{30}\times 0 + \frac{15}{30}\times 1 + \frac{9}{30}\times 2 + \frac{1}{30}\times 3 = \frac{36}{30} = \frac{6}{5}$$

$$E(X^2) = \frac{5}{30}\times 0^2 + \frac{15}{30}\times 1^2 + \frac{9}{30}\times 2^2 + \frac{1}{30}\times 3^2 = \frac{60}{30} = 2$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= 2 - \left[\frac{6}{5}\right]^2 = 2 - \frac{36}{25} = \frac{14}{25}$$

$$Standard\ deviation, \sigma = \sqrt{Var(X)} = \frac{\sqrt{14}}{5}$$

Page 75, Ex(4.1) No.4

Consider, Two tetrahedral are thrown

(1 , 1)	(1 , 2)	(1 , 3)	(1 , 4)
(2 , 1)	(2 , 2)	(2 , 3)	(2 , 4)
(3 , 1)	(3 , 2)	(3 , 3)	(3 , 4)
(4 , 1)	(4 , 2)	(4 , 3)	(4 , 4)

Let X be the sum of score

$$P_k = P(x = k)$$

$$P_2 = \frac{1}{16} \quad P_3 = \frac{2}{16} \quad P_4 = \frac{3}{16} \quad P_5 = \frac{4}{16} \quad P_6 = \frac{3}{16} \quad P_7 = \frac{2}{16} \quad P_8 = \frac{1}{16}$$

X	2	3	4	5	6	7	8
P(X)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
F (X)	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{6}{16}$	$\frac{10}{16}$	$\frac{13}{16}$	$\frac{15}{16}$	$\frac{16}{16}$

Pg 77, Ex4.2 ,No.1

If a man purchases a raffle ticket, he can win a prize of \$5000 or a second prize of \$2000 with probability 0.001 and 0.003. What should be a fair price to pay for ticket?

Fair price to pay for ticket

$$= 0.001 \times 5000 + 0.003 \times 2000 + 0.996 \times 0 = 11\,\$$$

Pg 77, Ex4.2,No.2

In a given business venture a man can make a profit of \$300 with probability 0.6 or take a loss of 100 with probability 0.4. Determine his expectation.

His expectation $= 0.6 \times 300 + 0.4 \times (-100) = 140$

Pg 77, Ex4.2 ,No. 3

Find (a) E (X) , (b) E (X^2), and (c) $E\left[\left(X - \overline{X}\right)^2\right]$ **for following probability.**

X	8	12	16	20	24
P (X)	1/8	1/6	3/8	1/4	1/12

$$E(X) = \frac{1}{8} \times 8 + \frac{1}{6} \times 12 + \frac{3}{8} \times 16 + \frac{1}{4} \times 20 + \frac{1}{12} \times 24$$

$$E(X)=\frac{1}{8}\times 8+\frac{1}{6}\times 12+\frac{3}{8}\times 16+\frac{1}{4}\times 20+\frac{1}{12}\times 24$$

$$E(X)=1+2+6+5+2=16$$

$$E(X^2)=\frac{1}{8}\times 8^2+\frac{1}{6}\times 12^2+\frac{3}{8}\times 16^2+\frac{1}{4}\times 20^2+\frac{1}{12}\times 24^2$$

$$E(X^2)=8+24+96+100+48=276$$

$$E(X-\overline{X})^2=[\tfrac{1}{8}(8-16)^2+\tfrac{1}{6}(12-16)^2+\tfrac{1}{4}(20-16)^2+\tfrac{1}{12}(24-16)^2]$$

$$\mathrm{E}[(\mathrm{X}-\overline{\mathrm{X}})^2]=8+\frac{8}{3}+4+\frac{16}{3}=20$$

Pg 77, Ex4.2 ,No.4

A bag contains two white balls and three black balls. Four persons A , B , C , D in the order named each draws one balls and does not replace it. The first to draw a white ball receives $10. Determine their expectations.

Expectation of A $= \frac{2}{5} \times 10$

Expectation of B $= \frac{3}{5} \times \frac{2}{4} \times 10$

Expectation of C $= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times 10$

Expectation of D $= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} \times 10$

Pg 79, Ex4.3 ,No.2

Two discs are draw, without replacement, for a box containing 3 red discs and 4 white discs. The discs are drawn at random. If X is the random variable 'the number for red discs drawn', find (a) E (X) , (b) the standard deviation of X.

X be the number of red

$R_X = \{0,1,2\}$ $\quad W = \text{White dic}$ $\quad R = \text{Red dic}$

$P(R,R) = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

$P(R,W) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$

$P(W,R) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

$P(W,W) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

$P_k = P(x = k)$

$P_0 = \frac{2}{7}$

$P_1 = \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$

$P_2 = \frac{1}{7}$

X	0	1	2
P(X)	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$
F (X)	$\frac{2}{7}$	$\frac{6}{7}$	$\frac{7}{7}$

$$E(X) = \frac{2}{7}\times 0 + \frac{4}{7}\times 1 + \frac{1}{7}\times 2 = \frac{6}{7}$$

$$E(X^2) = \frac{2}{7}\times 0^2 + \frac{4}{7}\times 1^2 + \frac{1}{7}\times 2^2 = \frac{8}{7}$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{8}{7} - \left[\frac{6}{7}\right]^2 = \frac{56-36}{49} = \frac{20}{49}$$

$$\sigma = \sqrt{\frac{20}{49}} = \frac{2\sqrt{5}}{7}$$

Pg 79, Ex4.3 ,No.3

The discrete random variable X has probability distribution shown below.

X	1	2	3	4
P (X)	3/8	1/8	1/4	1/4

Verify that $Var(2X + 3) = 4\,Var(X)$

X	1	2	3	4
P (X)	3/8	1/8	1/4	1/4

$$E[X] = \frac{3}{8}\times 1 + \frac{1}{8}\times 2 + \frac{1}{4}\times 3 + \frac{1}{4}\times 4 = \frac{3+2+6+8}{8} = \frac{19}{8}$$

$$E\left[X^2\right] = \frac{3}{8}\times 1^2 + \frac{1}{8}\times 2^2 + \frac{1}{4}\times 3^2 + \frac{1}{4}\times 4^2 = \frac{3+4+18+32}{8} = \frac{57}{8}$$

$$E[2X+3] = 2E[X] + 3 = 2\times\frac{19}{8} + 3 = \frac{31}{4}$$

$$E\left[(2X+3)^2\right] = E\left[4X^2 + 12X + 9\right] = 4E\left[X^2\right] + 12E[X] + 9 = 4\times\frac{57}{8} + 12\times\frac{19}{8} + 9$$

$$= \frac{57+57+18}{2} = \frac{132}{2} = 66$$

$$Var[X] = E[X^2] - [E(X)]^2 = \frac{57}{8} - \left(\frac{19}{8}\right)^2 = \frac{456-361}{64} = \frac{95}{64}$$

$$4\,Var[X] = 4\times\frac{95}{64} = \frac{95}{16}$$

$$Var[2X+3] = E[(2X+3)^2] - [E(2X+3)]^2 = 66 - \left(\frac{31}{4}\right)^2 = \frac{1056-961}{16} = \frac{95}{16}$$

$$\therefore\ Var[2X+3] = 4\,Var[X]$$

Pg 79 . EX (4.4) No. 1

Fine the cumulative distribution function for the ramdom variable where X is the score on an unbiased die.

X be the number of score

$$R_x = \{1,2,3,4,5,6\}$$

$$P(x=1)=\frac{1}{6} \quad P(x=2)=\frac{1}{6} \quad P(x=3)=\frac{1}{6} \quad P(x=4)=\frac{1}{6}$$

$$P(x=5)=\frac{1}{6} \quad P(x=6)=\frac{1}{6} \qquad F(X=i)=\frac{i}{6} \quad , i=1,2,3,4,5,6$$

X	1	2	3	4	5	6
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
F (X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{16}$	$\frac{5}{6}$	$\frac{6}{6}$

Pg 80, Ex4.4 ,No.3

For a discrete random variable X the cumulative distribution function F (X) is

X	1	2	3	4	5
F (X)	0.2	0.32	0.67	0.9	1

Find (a) $P(X = 3)$, (b) $P(X > 2)$

X	1	2	3	4	5
F(X)	0 .2	0 .32	0 .67	0 .9	1
P (X)	0 .2	0 .12	0 .35	0 .23	0 .1

(a) $P(x = 3) = 0.35$ (b) $P(x > 2) = 0.68$

Binomial Distribution

n = number of trials

p = probability of success in any single trial of n trials.

q = 1 – P = probability of failure in any single trial of n trials.

x = number of success in n trials.

n – x = number of failure in n trials.

$$P(x=k) = {}^{n}C_{k}\ p^{k}\ q^{n-k} \quad , \text{where } k = 0, 1, 2, \ldots.. n$$

This discrete probability distribution is often called the binomial distribution since for X = 0 , 1 , 2 , ….. , n it corresponds to successive terms in the binomial expansion

$$(p+q)^{n} = {}^{n}C_{0}\ p^{0}\ q^{n-0} + {}^{n}C_{1}\ p^{1}\ q^{n-1} + \ldots.. + {}^{n}C_{n}\ p^{n}\ q^{n-n}$$

Some Properties Binomial Distribution

$$\mu = E(X) = np$$

$$Var(X) = npq$$

$$\text{Standard deviation}, \sigma = \sqrt{Var(X)} = \sqrt{npq}$$

$$E(n - x) = nq$$

$$Var(n - x) = npq$$

Pg 84 , No.10

An ordinary die is thrown seven times. Find the probability of (i)obtaining exactly three six. (ii)obtain at most three six (iii) obtain exactly three not six(iv) obtain more than five time not six (v) expectation of getting six (vi) standard deviation

n = the numbers of thrown the die

p = the probability of getting six in any single thrown

q = the probability of not getting six in any single thrown

x = the numbers of times of getting six in n trials

n–x =the numbers of times of not getting six in n trials

$n = 7$ $\qquad p = \frac{1}{6}$ $\qquad q = \frac{5}{6}$

$n = 7$ $\qquad p = \frac{1}{6}$ $\qquad q = \frac{5}{6}$

$$P_k = P(x = k) = {}^{n}C_k \; p^k \; q^{n-k} \quad , k = 0,1,2,3,\ldots\ldots n$$

(i) obtain exactly three six

$$P(x = 3) = {}^{7}C_3 \; p^3 \; q^4 = {}^{7}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

$$= \frac{7\times6\times5}{3\times2\times1} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

$$= 35 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4$$

(ii) obtain at most three six

$$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= {}^{7}C_0 \; p^0 \; q^7 \;+ {}^{7}C_1 \; p^1 \; q^6 \;+ {}^{7}C_2 \; p^2 \; q^5 \;+ {}^{7}C_3 \; p^3 \; q^4$$

$$= 1 \times \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^6 + \frac{7 \times 6}{2 \times 1} \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5$$

$$+ \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5$$

$$= \left(\frac{5}{6}\right)^7 + 7 \times \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^6 + 21 \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5 + 35 \times \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^5$$

(iii) obtain exactly three not 6

$$P(7-x=3)=P(x=4)={}^{7}C_{4}\ p^{4}\ q^{3}={}^{7}C_{4}\ (\frac{1}{6})^{4}\ (\frac{5}{6})^{3}$$

$$=\frac{7\times 6\times 5}{3\times 2\times 1}\ (\frac{1}{6})^{4}\ (\frac{5}{6})^{3}$$

$$=35\ (\frac{1}{6})^{4}\ (\frac{5}{6})^{3}$$

(iv) obtain more than five time not six

$$P(7-x>5)=P(x<2)=P(x=0)+P(x=1)$$

$$= {}^{7}C_{0}\ p^{0}\ q^{7} + {}^{7}C_{1}\ p^{1}\ q^{6}$$

$$= 1\times\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{7} + 7\times\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{6}$$

$$= \left(\frac{5}{6}\right)^{7} + 7\times\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{6}$$

(v) expectation of getting six

$$\mu = E(X) = np$$

$$\mu = E(X) = 7 \times \frac{1}{6} = \frac{7}{6}$$

$$\text{(vi)Standard deviation, } \sigma = \sqrt{Var(X)} = \sqrt{npq}$$

$$\sigma = \sqrt{7 \times \frac{1}{6} \times \frac{5}{6}}$$

$$\sigma = \frac{\sqrt{35}}{6}$$

The probability that a pen drawn at random from a box of pens is defective is 0.1. If a sample of 6 pens is taken, find the probability that (i) no defective pen (ii) 5 or 6 defective pens (iii) less than 3 defective pen (iv) expectation of defective (v) expectative of non defective (vi) variance

n = the numbers of pens

p = the probability of getting defective pen in any single pen.

q = the probability of getting non defective pen in any single pen.

x = the no. of getting defective pen in 6 pen.

n – x = the no. of getting non defective pen in 6 pen.

$n = 6$ $p = 0.1$ $q = 0.9$

$$P_k = P(x = k) = {}^nC_k \, p^k \, q^{n-k} \quad , k = 0,1,2,3,\ldots\ldots n$$

$$\text{(i)} \; P(x = 0) = {}^6C_0 \, p^0 \, q^6 = 1 \times (0.1)^0 \, (0.9)^6 \quad = (0.9)^6$$

$$\text{(ii)} \; P(x = 5 \text{ or } 6) = P(x=5) + P(x=6)$$

$$= {}^6C_5 \, p^5 \, q^1 \quad + {}^6C_6 \, p^6 \, q^0$$

$$= {}^6C_1 \, p^5 \, q^1 \quad + {}^6C_0 \, p^6 \, q^0$$

$$= 6 \times (0.1)^6 \, (0.9)^1 \quad + 1 \times (0.1)^6 \, (0.9)^0$$

$$= 6 \times (0.1)^6 \, (0.9) \; + (0.1)^6$$

(iii) $P(X < 3) = P(X = 0) + P(X=1) + P(X=2)$

$$= {}^6C_0\ p^0\ q^6 + {}^6C_1\ p^1\ q^5 + {}^6C_2\ p^2\ q^4$$

$$= 1 \times (0.1)^0\ (0.9)^6 + 6 \times (0.1)^1\ (0.9)^5 + \frac{6\times5}{2\times1} \times (0.1)^2\ (0.9)^4$$

$$= (0.9)^6 + 6 \times (0.1)\ (0.9)^5 + 15 \times (0.1)^2\ (0.9)^4$$

$$\mu = E(\text{defective}) = 6\times0.1=0.6$$

$$\mu = E(\text{non defective}) = 6\times0.9=5.4$$

$$\text{Variance} = npq = 6\times(0.1)\,(0.9) = 0.54$$

$$\sigma = \sqrt{0.54}$$

The probability that a person serves a company A is 0.6. Find the probability that in a randomly selected sample of 8 labourers there are (i) exactly 3 who serve company A. (ii) more than 5 who serve company A. (iii)less than 2 who does not serve company A (iv)expectation of serve company A (v)standard deviation

n = the numbers of labourers

p = the probability of serve copany A in any single person

q = the probability of not serve copany A in any single person

x = the numbers of person who serve company A in n labourers

n – x = the numbers of person who do not serve company A in n labourers

$n = 8$ $p = 0.6$ $q = 0.4$

$$P_k = P(x = k) = {}^nC_k\ p^k\ q^{n-k} \quad , k = 0,1,2,3,\ldots\ldots n$$

(i) exactly 3 person who serve company A

$$P(x = 3) = {}^8C_3\ p^3\ q^5 = {}^8C_3\ (0.6)^3\ (0.4)^5$$

$$= \frac{8\times7\times6}{3\times2\times1}(0.6)^3\ (0.4)^5$$

$$= 56\ (0.6)^3\ (0.4)^5$$

(ii) more than 5 who serve company A

$$P(x>5) = P(x=6)+P(x=7)+P(x=8)$$

$$= {}^8C_6\ p^6\ q^2 + {}^8C_7\ p^7\ q^1 + {}^8C_8\ p^8\ q^0$$

$$= {}^8C_2\ p^6\ q^2 + {}^8C_1\ p^7\ q^1 + {}^8C_0\ p^8\ q^0$$

$$= \frac{8\times 7}{2\times 1}\times (0.6)^6\ (0.4)^2 + 8\times (0.6)^7\ (0.4)^1 + 1\times (0.6)^8\ (0.4)^0$$

$$= 28\times (0.6)^6\ (0.4)^2 + 8\times (0.6)^7\ (0.4) + (0.6)^8$$

(iii) less than 2 who does not serve company A

$$P(8-x<2)=P(x>6)=P(x=7)+P(x=8)$$

$$= {}^{8}C_{7}\ p^{7}\ q^{1} + {}^{8}C_{8}\ p^{8}\ q^{0}$$

$$= {}^{8}C_{1}\ p^{7}\ q^{1} + {}^{8}C_{0}\ p^{8}\ q^{0}$$

$$= 8\,(0.6)^{7}\,(0.4)^{1} + 1\times(0.6)^{8}\,(0.4)^{0}$$

$$= 8\,(0.6)^{7}\,(0.4) + (0.6)^{8}$$

(iv) expectation of serve company A

$$\mu = E(X) = np$$

$$\mu = E(X) = 8 \times 0.6 = 4.8$$

$$\text{Standard deviation},\ \sigma = \sqrt{Var(X)} = \sqrt{npq}$$

$$\sigma = \sqrt{8 \times 0.6 \times 0.4}$$

Pg 84 , No.14

In a group of people the expected number who wear glasses is 2 and the variance is 1.6. Find the probability that (i) a person chosen at random from the group wear glasses, (ii) 6 people in a group wear glasses.

n = the numbers of people in a group

p = the probability of wear glasses in single person

q = the probability of not wear glasses in single person

x = the no. of persons who wear glasses in a group

n–x =the no. of person who does not wear glasses in a group

$n = n$ $\quad p = p$ $\quad q = q$ $\quad E(X) = np = 2$ $\quad \sigma = npq = 1.6$

$$E(X) = np = 2$$

$$np = 2 \quad \_\_\_\_\_\_ \quad (1)$$

$$Var(X) = npq = 1.6$$

$$npq = 1.6 \quad \_\_\_\_\_\_ \quad (2)$$

$$eq(2) \div eq(1) \Rightarrow \frac{npq}{np} = 0.8$$

$$q = 0.8$$

$$p + q = 1$$

$$p + 0.8 = 1$$

$$p = 0.2$$

The probability that a person chosen at random from the group wear glasses is 0.2

(ii) $P(X=6) = {}^{10}C_6 \; p^6 \; q^4$

$$= \frac{10\times 9\times 8\times 7\times 5\times 4}{6\times 5\times 4\times 3\times 2\times 1}\times (0.2)^6 \; (0.8)^4$$

$$= 210\times (0.2)^6 \; (0.8)^4$$

(iii) $P(7-x=3) = P(X=4) = {}^7C_4\ p^4\ q^3 = {}^7C_4\ (\frac{1}{6})^4\ (\frac{5}{6})^3$

$$= \frac{7\times6\times5}{3\times2\times1}\ (\frac{1}{6})^4\ (\frac{5}{6})^3$$

$$= 35\ (\frac{1}{6})^4\ (\frac{5}{6})^3$$

Pg 83 , No.3

In a multiple choice test there are 10 questions and for each question there is a choice of 4 answers, only one of which is correct. If a student guesses at each of the answers, find the probability that he gers (a) no correct (b) more than 7 correct (c) more than 2 correct.

n = the numbers of questions

p = the probability of correct in any single question

q = the probability of incorrect in any single question

x = the no. of correct answers in n questions.

n–x =the no. of incorrect answers in n questions

$\mathrm{n} = 10 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

$n = 10 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

(i) no correct

$$P(x=0) = {}^{10}C_0\ p^0\ q^{10} = \left(\frac{3}{4}\right)^{10}$$

(ii) more than 7 correct

$$P(x>7) = P(x=8)+P(x=9)+P(x=10)$$

$$= {}^{10}C_8\ p^8\ q^2 + {}^{10}C_9\ p^9\ q + {}^{10}C_{10}\ p^{10}\ q^0$$

$$= 45\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 + 10\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)^{10}$$

(iii) more than 2 correct

$$P(X>2) = 1-\{P(X\le 2\} = 1-\{P(X=0)+P(X=1)+P(X=2)\}$$

$$= 1-\{{}^{10}C_0\ p^0\ q^{10} + {}^{10}C_1\ p^1\ q^9 + {}^{10}C_2\ p^2\ q^8\}$$

$$= 1-\{\left(\frac{3}{4}\right)^{10} + 10\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^9 + 45\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^8\}$$

(iv) only three consecutive correct

$$P(\text{only three consecutive correct}) = 8 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^4$$

Pg 83 , No.7

A surgical technique is performed on seven patients. You are told there is a 70% chance of success, Find the probability that the surgery is successful for (a) exactly five patient

(b) at least five patients (c) less than five patients,

n = the numbers of patients

p = the probability of success in any single patient

q = the probability of fail in any single patient

x = the no. of success in n patients.

n–x =the no. of fail in n patients

$n = 7 \qquad p = \frac{7}{10} \qquad q = \frac{3}{10}$

$n = 7$ $p = \frac{7}{10}$ $q = \frac{3}{10}$

(i) successful for exactly five patient

$$P(x=5) = {}^7C_5\ p^5\ q^2 = 21\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)^2$$

(ii) successful for at least five patient

$$P(x \geq 5) = P(x=5)+P(x=6)+P(x=7)$$

$$= {}^7C_5\ p^5\ q^2 + {}^7C_6\ p^6\ q + {}^7C_7\ p^7\ q^0$$

$$= 21\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)^2 + 7\left(\frac{7}{10}\right)^6\left(\frac{3}{10}\right) + \left(\frac{7}{10}\right)^7$$

(iii) successful for less than five patient

$$P(x < 5) = 1-\{P(x \geq 5\} = 1 - \{P(x=5)+P(x=6)+P(x=7)\}$$

$$= 1 - \{ {}^7C_5\ p^5\ q^2 + {}^7C_6\ p^6\ q + {}^7C_7\ p^7\ q^0 \}$$

$$= 1 - \left\{ 21\left(\frac{7}{10}\right)^5\left(\frac{3}{10}\right)^2 + 7\left(\frac{7}{10}\right)^6\left(\frac{3}{10}\right) + \left(\frac{7}{10}\right)^7 \right\}$$

Pg 83 , No.9

One in four adults is currently on a diet. In a room sample of eight adults, what is the probability that the currently on a diet is (a) exactly three (b) at least three, and (c) more than three ?

n = the numbers of adults

p = the probability of diet in any single adult

q = the probability of no diet in any single adult

x = the no. of diet in n adults.

n–x =the no. of no diet in n adult

$n = 8 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

$$P_k = P(x = k) = {}^{n}C_k \; p^k \; q^{n-k} \quad , k = 0,1,2,3,\ldots\ldots n$$

$n = 8 \qquad p = \frac{1}{4} \qquad q = \frac{3}{4}$

(i) diet exactly three

$$P(x = 3) = {}^{8}C_{3}\ p^{3}\ q^{5} = 56\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5}$$

(ii) diet at least three

$$P(x \geq 3) = 1 - P(x < 3) = 1 - \{\ P(x=0) + P(x=1) + P(x=2)\ \}$$

$$= 1 - \left\{\ {}^{8}C_{0}\ p^{0}\ q^{8} + {}^{8}C_{1}\ p\ \ q^{7} + {}^{8}C_{2}\ p^{2}\ q^{6}\ \right\}$$

$$= 1 - \left\{\ \left(\frac{3}{4}\right)^{8} + 8\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{7} + 28\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{6}\ \right\}$$

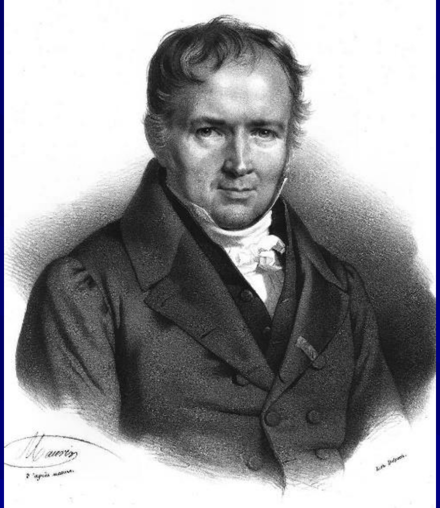
(iii) diet more than three

$$P(x > 3) = 1 - P(x \leq 3) = 1 - \{\ P(x=0) + P(x=1) + P(x=2) + P(x=3)\ \}$$

$$= 1 - \left\{\ {}^{8}C_{0}\ p^{0}\ q^{8} + {}^{8}C_{1}\ p\ \ q^{7} + {}^{8}C_{2}\ p^{2}\ q^{6} + {}^{8}C_{3}\ p^{3}\ q^{5}\ \right\}$$

$$= 1 - \left\{\ \left(\frac{3}{4}\right)^{8} + 8\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^{7} + 28\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{6} + 56\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5}\ \right\}$$

Poison Distribution



Poison Distribution

When n is larg and p is small in a binomial distribution.

If x is the numbers of occurrences of random event in an interval of time or space or some volume of matter. The random variavle x has a Poisson distribution if and only if the probability distribution is given by

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad , x = 0,1,2,......$$

Where, $\lambda = n\,p$

$$P(x = k) = \frac{e^{-\lambda} \lambda^{k}}{k!} \quad , k = 0,1,2,......$$

Poison Distribution

Examples of events which might follow a poisson distribution distribution:

The number of

(i) flaws in a given length of material

(ii) car accidents on a particular stretch of road in one day

(iii) accidents in a factory in one week

(iv) telephone calls made to a switchboard in a given time

(v) insurance claim made to a company in a given time

(vi) particles emitted by a radioactive source in a given time

A time interval

$$P(x=k)=\frac{e^{-\lambda t}(\lambda t)^k}{k!} \quad , k=0,1,2,......$$

A given length

$$P(x=k)=\frac{e^{-\lambda l}(\lambda l)^k}{k!} \quad , k=0,1,2,......$$

A given region

$$P(x=k)=\frac{e^{-\lambda a}(\lambda a)^k}{k!} \quad , k=0,1,2,......$$

A given volume

$$P(x=k)=\frac{e^{-\lambda v}(\lambda v)^k}{k!} \quad , k=0,1,2,......$$

Using the Poison Distribution as an approximation to the Binomial Distribution

A binomial distribution with parameter n and p can be approximated by a poisson distribution, with parameter $\lambda = np$, if n is large (> 50 say) and p is small (< 0.1 say). The approximation get better as $n \rightarrow \infty$ and $p \rightarrow 0$.

$$(p + q)^n \rightarrow e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots\ldots\ldots \right)$$

$$When \quad \lambda = n\,p$$

$$P(x = k) = \frac{e^{-\lambda} \; \lambda^k}{k!} \quad , k = 0,1,2,\ldots\ldots$$

Find the probability that at least double sixes are obtained when two dice are thrown 90 times. ($e^{-2.5} = 0.082$)

Throw two dice, P (double sixes) $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$X \sim Bin(90, \frac{1}{36})$ *and* $n\,p = 90 \times \frac{1}{36} = 2.5$, $X \sim Poi(2.5)$

$$P(x = k) = \frac{e^{-2.5} (2.5)^k}{k!} \quad , k = 0,1,2,3,4,5,\ldots.$$

$$P(x \ge 2) = 1 - \{ P(x=0) + P(x=1) \}$$

$$= 1 - \left(\frac{e^{-2.5} (2.5)^0}{0!} + \frac{e^{-2.5} (2.5)^1}{1!} \right)$$

$$= 1 - e^{-2.5} (1 + 2.5) \quad = 1 - (0.082)(3.5) = 0.713$$

By the Binomial theorem

$$\left(1+\frac{x}{n}\right)^n = 1+n\left(\frac{x}{n}\right)+\frac{n(n-1)}{2!}\left(\frac{x^2}{n^2}\right)+\frac{n(n-1)(n-2)}{3!}\left(\frac{x^3}{n^3}\right)+\ldots\ldots$$

$$= 1 + x + \frac{x^2}{2!}\left(\frac{n(n-1)}{n^2}\right) + \frac{x^3}{3!}\left(\frac{n(n-1)(n-2)}{n^3}\right)+\ldots\ldots$$

$$= 1 + x + \frac{x^2}{2!}\left(1-\frac{1}{n}\right) + \frac{x^3}{3!}\left(\frac{(n-1)}{n}\frac{(n-2)}{n}\right)+\ldots\ldots$$

Now, $n \to \infty$ *then* $\left(1-\frac{1}{n}\right) \to 1$

$$\left(1+\frac{x}{n}\right)^n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots = e^x$$

i.e, $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n \to e^x$ *Similarly* $\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n \to e^{-x}$

$$q = 1 - p \quad and \quad p = \frac{n\,p}{n} = \frac{\lambda}{n} \qquad then \quad q = 1 - \frac{\lambda}{n}$$

$$\left(q + \frac{\lambda}{n} \right)^n$$

$$= \left(1 - \frac{\lambda}{n}\right)^n + n\left(1 - \frac{\lambda}{n}\right)^{n-1}\left(\frac{\lambda}{n}\right) + \left(\frac{n(n-1)}{2!}\right)\left(1 - \frac{\lambda}{n}\right)^{n-2}\left(\frac{\lambda}{n}\right)^2$$

$$+ \left(\frac{n(n-1)(n-2)}{3!}\right)\left(1 - \frac{\lambda}{n}\right)^{n-3}\left(\frac{\lambda}{n}\right)^3 + \ldots\ldots.$$

$$= \left(1 - \frac{\lambda}{n}\right)^n \left[1 + \frac{n}{\left(1 - \frac{\lambda}{n}\right)}\left(\frac{\lambda}{n}\right) + \frac{n(n-1)}{2!\left(1 - \frac{\lambda}{n}\right)^2}\left(\frac{\lambda}{n}\right)^2 + \ldots\ldots \right]$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{\left(1 - \frac{\lambda}{n}\right)} + \frac{\lambda^2\left(1 - \frac{1}{n}\right)}{2!\left(1 - \frac{\lambda}{n}\right)^2} + \ldots\ldots \right], \quad then \quad n \to \infty$$

$$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \ldots\ldots\ldots \right]$$

Expectation and Variance

X	0	1	2	3 ....
P (X)	$e^{-\lambda}$	$e^{-\lambda}\lambda$	$e^{-\lambda}\frac{\lambda^2}{2!}$	$e^{-\lambda}\frac{\lambda^3}{3!}$

$$E(X) = 0 \times e^{-\lambda} + 1 \times e^{-\lambda}\lambda + 2 \times e^{-\lambda}\frac{\lambda^2}{2!} + 3 \times e^{-\lambda}\frac{\lambda^3}{3!} + \ldots\ldots\ldots$$

$$= \lambda e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots\ldots\right)$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda$$

$$E(X^2) = 0^2 \times e^{-\lambda} + 1^2 \times e^{-\lambda}\lambda + 2^2 \times e^{-\lambda}\frac{\lambda^2}{2!} + 3^2 \times e^{-\lambda}\frac{\lambda^3}{3!} + \ldots..$$

$$= \lambda e^{-\lambda}\left(1 + 2\lambda + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} \ldots\ldots\right)$$

$$= \lambda e^{-\lambda}\left(1 + (\lambda + \lambda) + \left(\frac{\lambda^2}{2!} + \frac{2\lambda^2}{2!}\right) + \left(\frac{\lambda^3}{3!} + \frac{3\lambda^3}{3!}\right) + \ldots\ldots\right)$$

$$= \lambda e^{-\lambda}\left[\left(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\ldots\right) + \left(\lambda + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \ldots\ldots\ldots\right)\right]$$

$$= \lambda e^{-\lambda}\left(e^{\lambda} + \lambda\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots\ldots\ldots\right)\right)$$

$$= \lambda e^{-\lambda}\left(e^{\lambda} + \lambda e^{\lambda}\right)$$

$$= \lambda e^{-\lambda} e^{\lambda}(1 + \lambda) = \left(\lambda + \lambda^2\right)$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= (\lambda + \lambda^2) - \lambda^2$$

$$Var(X) = \lambda$$

Mode of Poisson

In general, λ is not an integer, then mode is the integer

$$\lambda - 1 < \text{mode} < \lambda$$

If the number of bacterial colonies on a pertri dish follows a Poisson distribution with average number 2.5 per cm^2 , find the probability that (a) in 1 cm^2 there will be no bacterial colonies (b) in 1 cm^2 there will be more than 4 bacterial colonies (c) in 2 cm^2 there will be 4 bacterial colonies (d) in 4 cm^2 there will be 6 bacterial colonies. ($e^{-2.5} = 0.082$) ($e^{-5} = 0.0067$) ($e^{-10} = 4.5 \times 10^{-5}$)

$$\lambda = 2.5 \ per \ cm^2$$

$$P_k = P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!} \quad , k = 0,1,2,3,4,5,....$$

x be the the number of bacterial colonies

$$(a) \ P(x = 0) = \frac{e^{-2.5} (2.5)^0}{0!} = 0.082$$

$$(a)\ P(x=0)=\frac{e^{-2.5}\ (2.5)^{0}}{0!}=0.082$$

$$(b)\ P(x>4)=1-\{P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)\}$$

$$=1-\{\frac{e^{-2.5}\ (2.5)^{0}}{0!}+\frac{e^{-2.5}\ (2.5)^{1}}{1!}+\frac{e^{-2.5}\ (2.5)^{2}}{2!}+\frac{e^{-2.5}\ (2.5)^{3}}{3!}+\frac{e^{-2.5}\ (2.5)^{4}}{4!}\}$$

$$=1-e^{-2.5}\left\{1+2.5+\frac{(2.5)^{2}}{2}+\frac{(2.5)^{3}}{6}+\frac{(2.5)^{4}}{24}\right\}$$

$$(c)\ \mu=\lambda a=2.5\times 2=5$$

$$P(x=4)=\frac{e^{-5}\ (5)^{4}}{4!}=\frac{0.067\times 625}{4!}=1.745$$

(d) $\mu = \lambda a = 2.5 \times 4 = 10$

$$P(x = 6) = \frac{e^{-10}\,(10)^6}{6!} = \frac{4.5 \times 10^{-5} \times 10^6}{6!} = 0.0139$$

Suppose that the probability that a certain type of inoculation take effect is 0.995. What is the probability at most two out of 400 people given the inoculation, find that it has no taken effect ? ($e^{-2} = 0.1315$)

$$n = 400 ,$$

$$P(\textit{inoculation taken effect}) = 0.995$$

$$P(\textit{inoculation taken no effect}) = 0.005$$

$$\lambda = n\ p = 400 \times 0.005 = 2$$

$$P_k = P(x = k) = \frac{e^{-\lambda}\ \lambda^k}{k!}\ ,\ k = 0,1,2,3,4,\ldots\ldots$$

x be the the number of people among the 400 from who the inoculation does not effect

$$P(x \le 2) = P(x = 0) + P(x=1) + P(x=2)$$

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= \frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!}$$

$$= e^{-2}\left[1 + 2 + 2\right]$$

$$= 0.1353 \times 5$$

$$= 0.6765$$

The expected number of people among the 1000 from who the inoculation does not effect $= 1000 \times 0.005 = 5$

In a certain region the number of persons who became seriously ill each year eating a certain poisonous plant is a random variable had being the Poission distribution with mean is 2. What is the probability of at most 3 such illnesses in a given year. ($e^{-2} = 0.1315$)

Solution

x be the the number of person who become seriously ill each year from eating a certain poisonuous plant

$$\lambda = 2$$

$$P_k = P(x = k) = \frac{e^{-\lambda}\, \lambda^k}{k!} \quad , k = 0,1,2,3,4,5\ldots$$

$$P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= \frac{e^{-2} \times 2^0}{0!} + \frac{e^{-2} \times 2^1}{1!} + \frac{e^{-2} \times 2^2}{2!} + \frac{e^{-2} \times 2^3}{3!}$$

$$= e^{-2} \left[1 + 2 + 2 + 1.33\right]$$

$$= 0.1353 \times 6.33$$

$$= 0.8564$$

The probability that a car will have a flat tire while driving over a certain bridge is 0.0002. Find the probability that among 2000 cars driven over the bridge not more than one will have a flat tire. ($e^{-0.4} = 0.67$)

Solution

x be the the number of car which will have flat tire while driving over a certain bridge

$$n = 2000, \quad P(\textit{flat tire while driving over}) = 0.0002$$

$$\lambda = 2000 \times 0.0002 = 0.4$$

$$P_k = P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \; k = 0,1,2,3,4,\ldots\ldots$$

$$P(x \leq 1) = P(x = 0) + P(x = 1)$$

$$P(x \le 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-0.4} \times (0.4)^0}{0!} + \frac{e^{-0.4} \times (0.4)^1}{1!}$$

$$= e^{-0.4} \left[1 + 0.4\right]$$

$$= 0.67 \times 1.4$$

$$= 0.938$$

The evarage car will have a flat tire while driving over a certain bridge is 0.4 per day. Find the expected number of the day out of 100 days when there will be (i) no flat tire (ii) nor more than one flat tire (iii) between 2 and 5 flat tire.
. ($e^{-0.4} = 0.67$)

Solution

x be the the number of car which will have flat tire

$$\lambda = 0.4$$

$$P_k = P(x = k) = \frac{e^{-\lambda} \lambda^k}{k!} , k = 0,1,2,3,4,......$$

$$(i)\ P(x = 0) = \frac{e^{-0.4} (0.4)^k}{0!} = 0.67$$

The expect no. of day which no flat tire

$$= 0.67 \times 100 = 67\ days$$

$$(ii)\ P(x \le 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-0.4} \times (0.4)^{0}}{0!} + \frac{e^{-0.4} \times (0.4)^{1}}{1!}$$

$$= e^{-0.4}\left[1 + 0.4\right]$$

$$= 0.67 \times 1.4$$

$$= 0.938$$

The expect no. of day which not more one flat tire

$$= 0.938 \times 100 = 93.8 = 94\ \textit{days}$$

$$(ii)\ P(2 < x < 5) = P(x=3) + P(x=4)$$

$$= \frac{e^{-0.4} \times (0.4)^3}{3!} + \frac{e^{-0.4} \times (0.4)^4}{4!}$$

$$= e^{-0.4}\left[0.0106 + 0.0011\right]$$

$$= 0.67 \times 0.0117$$

$$= 0.0078$$

The expect no. of day which between 2 and 5 flat tires

$$= 0.0078 \times 100 = 0.78 = 1\ day$$

Flaws in the planting of target sheet of metal occurs at random. On the average of one in each section of area 10 square feet. What is the probability that a 5 by 8 will have (i) no flaws ? (ii) at most one flaws ? ($e^{-4} = 0.0183$)

Solution

x be the number of flaws

$$\lambda = \frac{1}{10} = 0.1 \qquad a = 5 \times 8 = 40$$

$$\mu = \lambda a = 0.1 \times 40 = 4$$

$$P_k = P(x = k) = \frac{e^{-\lambda a} (\lambda a)^k}{k!} , k = 0,1,2,3,....$$

$$(i)P(x=0) = \frac{e^{-4} \times (4)^{0}}{0!} = 0.0183$$

$$(ii)P(x \le 1) = P(x=0) + P(x=1)$$

$$= \frac{e^{-4} \times (4)^{0}}{0!} + \frac{e^{-4} \times (4)^{1}}{1!}$$

$$= e^{-4} \times [1+4]$$

$$= 0.0183 \times 5$$

$$= 0.0915$$

Failure of electron taken airborn applications have been found to follow clearly poisson postulates. A recever with sixtten tubes suffers. A tube failure on the average of one every 50 hours of operating time. (i) What is the probability of more than one failure on an 8 hours missin ? (ii) What is the expected number of failures in 1000 hours of operation time ? ($e^{-0.16} = 0.8521$)

Solution

x be the the number of tube failure on the given time

$$\lambda = \frac{1}{50} = 0.02 \qquad t = 8$$

$$\mu = \lambda t = 0.02 \times 8 = 0.16$$

$$P_k = P(x = k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}, \; k = 0, 1, 2, 3, \ldots\ldots$$

$$(i)\, P(x>1) = 1 - \{\, P(x=0) + P(x=1) \}$$

$$= 1 - \left[\frac{e^{-0.16}\,(0.16)^0}{0!} + \frac{e^{-0.16}\,(0.16)^1}{1!} \right]$$

$$= 1 - e^{-0.16}\,[1 + 0.16\,]$$

$$= 1 - 0.8521 \times 1.16$$

$$= 0.0116$$

$$(ii)\ \lambda = \frac{1}{50} = 0.02 \qquad ,\ t = 1000$$

$$\mu = \lambda\, t = 0.02 \times 1000 = 20$$

Expected number of failure in 1000 hours is 20

A book containing 750 pages has 500 misprints. Assume that the misprints occur at random, find the probability that a particular page contain (i) no misprint (ii) exactly 4 misprints (iii) more than 2 misprints. ($e^{-0.67} = 0.512$)

Solution

x be the number of misprints in a particular page

$$\lambda = \frac{500}{750} = 0.67$$

$$P_k = P(x = k) = \frac{e^{-\lambda} (\lambda)^k}{k!}, \; k = 0, 1, 2, 3, 4, \ldots\ldots$$

$$(i)\ P(x=0) = \frac{e^{-0.67} \times (0.67)^0}{0!} = 0.512$$

$$(ii)\ P(x=4) = \frac{e^{-0.67} \times (0.67)^4}{4!} = 0.512 \times 0.008 = 0.0041$$

$$(iii)\ P(x>2) = 1 - \{ P(x=0) + P(x=1) + P(x=2) \}$$

$$= 1 - \left[\frac{e^{-0.67}\ (0.67)^0}{0!} + \frac{e^{-0.67}\ (0.67)^1}{1!} + \frac{e^{-.67}\ (0.67)^2}{2!} \right]$$

$$= 1 - e^{-0.67} \left[1 + 0.67 + 0.22 \right]$$

$$= 1 - 0.512 \times 1.89 = 0.0323$$

(iv) How many pages which no misprints in the book

The no. of pages which no misprints in the book

$$= 0.512 \times 750 = 384$$

(v) What is the probability that one misprints in each particular two pages ?

$$P(x = 1) = \frac{e^{-0.67} \times (0.67)^1}{1!} = 0.3430$$

$$P(\text{one misprint in each particular two pages}) = P_1 \times P_1$$

$$= 0.3430 \times 0.3430$$

$$= 0.1176$$

An insurance company recieves on average 2 claims per week from the certain factory. Assuming that the number of claims follows a Poisson distribution, find the probabimity that (i) it recieves more than 3 claims in a given week (ii) it recieves more than 2 claims in a given forthnight (iii) it recieves no claims on a given days, assuming that the factory operates on a 5 days week.? ($e^{-2} = 0.1353$) ($e^{-4} = 0.0183$) ($e^{-0.4} = 0.67$)

Solution

x be the the number of claimn in the given week

$$\lambda = 2$$

$$P_k = P(x = k) = \frac{e^{-\lambda} (\lambda)^k}{k!} \quad , k = 0,1,2,3,\ldots\ldots$$

$$(i)\, P(x > 3) = 1 - \{ P(x=0) + P(x=1) + P(x=2) + P(x=3) \}$$

$$= 1 - \left[\frac{e^{-2}\,(2)^0}{0!} + \frac{e^{-2}\,(2)^1}{1!} + \frac{e^{-2}\,(2)^2}{2!} + \frac{e^{-2}\,(2)^3}{3!} \right]$$

$$= 1 - e^{-2}\left[1 + 2 + 2 + 1.3333\right]$$

$$= 1 - 0.1353 \times 6.3333$$

$$= 0.1432$$

$(ii)\ \mu = \lambda t = 2 \times 2 = 4$

$$P(x > 2) = 1 - \{ P(x=0) + P(x=1) + P(x=2) \}$$

$$= 1 - \left[\frac{e^{-4}\,(4)^0}{0!} + \frac{e^{-4}\,(4)^1}{1!} + \frac{e^{-4}\,(4)^2}{2!} \right]$$

$$= 1 - e^{-4}\left[1 + 4 + 8\right]$$

$$= 1 - 0.0183 \times 13$$

$$= 0.7618$$

$$(iii)\ \mu = \lambda t = 2 \times \frac{1}{5} = 0.4$$

$$P(x = 0) = \frac{e^{-0.4}\,(0.4)^{0}}{0!} = 0.67$$