Continuous Probability Distribution

If X is a continuous random variable with probability density function f (x) valid over the range

$$a \le x \le b \ \ then$$

$$P(a \le x \le b) = \int_a^b f(x)\,dx = 1$$

Mathematical Expectation

$$\mu = E(X) = \int_a^b f(x).\, x \; dx$$

$$E(X^2) = \int_a^b f(x).\, x^2 \; dx$$

Some Results 0f Expectation

$$a \text{ and } b \text{ are constant}$$

$$\text{(i)} \quad E(a) = a$$

$$\text{(ii)} \quad E[ax] = aE[x]$$

$$\text{(iii)} \quad E[ax+b] = aE[x]+b$$

$$\text{(iv)} \quad E[f(x) \pm g(x)] = E[f(x)] \pm E[g(x)]$$

Variance

$$Var(X) = E(x^2) - \{E(x)\}^2$$

Standard deviation

$$\sigma = \sqrt{Var(x)}$$

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

Some Results Of Variance

$$a \text{ and } b \text{ are constant}$$

$$(i) \quad Var(a) = 0$$

$$(ii) \quad Var[ax] = a^2 \, Var[x]$$

$$(iii) \quad Var[ax + b] = a^2 \, Var[x]$$

$$(iv) \quad Var[f(x) \pm g(x)] = Var[f(x)] \pm Var[g(x)]$$

Cumulative Distribution Function

If X is a continuous random variable with probability density function f (x) define for a < x < b then the cumulative distribution function is given by F (t) where

$$F(t) = P(a \leq x \leq t) = \int_{a}^{t} f(x)\,dx$$

$$F(x) = P(a \leq x \leq x) = \int_{a}^{x} f(x)\,dx$$

Median

The median splits the area under the curve y = f (x) into two halves. So if the value of the median is m ,

$$P(a \le x \le m) = \int_a^m f(x)\,dx = \frac{1}{2} = 0.5$$

$$\text{i.e ,} \quad F(m) = \frac{1}{2}$$

A continuous random variable X has probability density function f (x) where

$$f(x)=\begin{cases} k(x+2)^2 & , -2 \le x < 0 \\ 4k & , 0 \le x \le 1\frac{1}{3} \\ 0 & , \text{otherwise} \end{cases}$$

(a)find the value of the constant k

(b)sketch y = f (x)

(c)find $P(-1 \le x \le 1)$

$$\int_{-2}^{0} k(x+2)^2\,dx + \int_{0}^{\frac{4}{3}} 4k\,dx = 1$$

$$\int_{-2}^{0} k(x+2)^2\,dx + \int_{0}^{\frac{4}{3}} 4k\,dx = 1$$

$$k\left[\frac{(x+2)^3}{3}\right]_{-2}^{0} + 4k\left[x\right]_{0}^{\frac{4}{3}} = 1$$

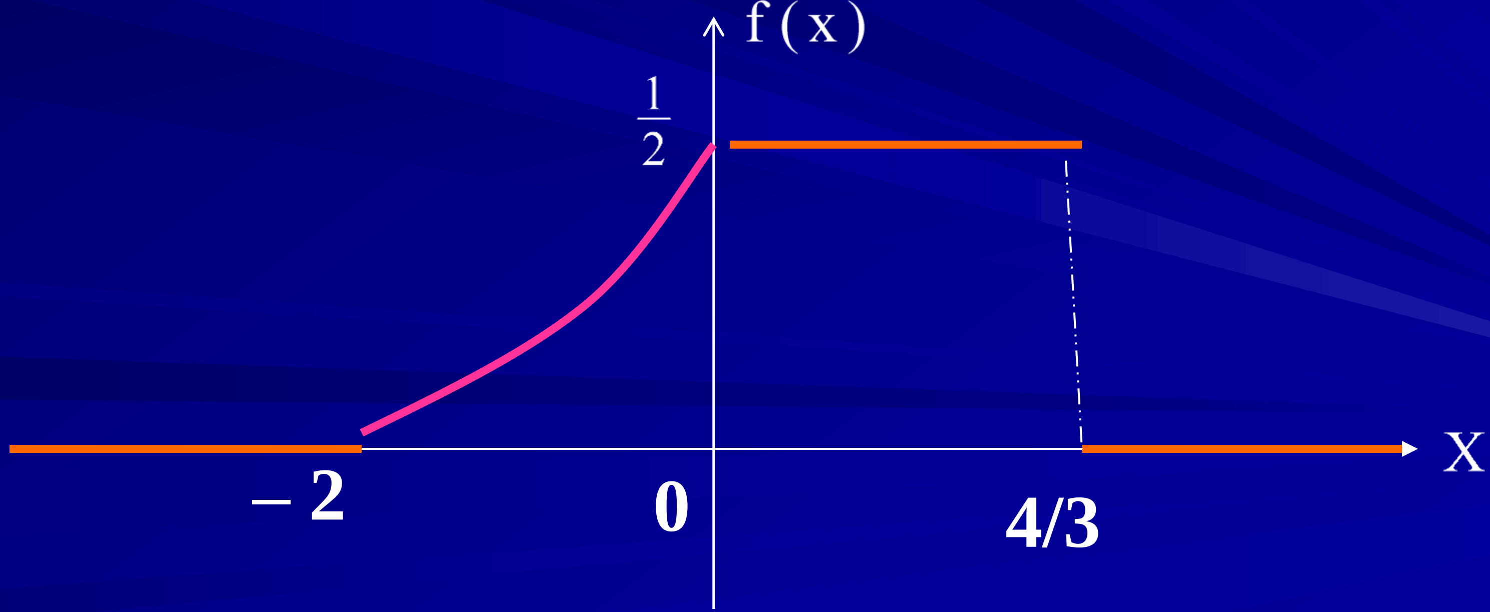
$$\frac{8}{3}k + \frac{16}{3}k = 1$$

$$k = \frac{1}{8}$$

$$f(x)=\begin{cases}\frac{1}{8}(x+2)^2 & , \quad -2 \le x < 0\\ \frac{1}{2} & , \ 0 \le x \le 1\frac{1}{3}\\ 0 & , \ \text{otherwise}\end{cases}$$

$f(-2)=0$ $\qquad f(0)=\frac{1}{2}$ $\qquad f(0)=\frac{1}{2}$ $\qquad f(1\frac{1}{3})=\frac{1}{2}$

$(-2\ ,\ 0)$ $\qquad (0\ ,\ \frac{1}{2})$ $\qquad (0\ ,\ \frac{1}{2})$ $\qquad (\frac{4}{3}\ ,\ \frac{1}{2})$



$$P(-1 \le x \le 1) = \int_{1}^{0} \frac{1}{8}(x+2)^2\,dx + \int_{0}^{1} \frac{1}{2}\,dx$$

$$= \frac{1}{24}\left[(x+2)^3\right]_{-1}^{0} + \frac{1}{2}\left[x\right]_{0}^{1}$$

$$= \frac{1}{24}\left[8 - 1\right] + \frac{1}{2}\left[1 - 0\right]$$

$$= \frac{19}{24}$$

$$E(X)=\int_{-2}^{0}\frac{1}{8}(x+2)^2 x\,dx+\int_{0}^{\frac{4}{3}}\frac{1}{2}x\,dx$$

$$=\frac{1}{8}\int_{-2}^{0}(x^3+4x^2+4x)\,dx+\frac{1}{2}\int_{0}^{\frac{4}{3}}x\,dx$$

$$=\frac{1}{8}\left[\frac{x^4}{4}+\frac{4x^3}{3}+2x^2\right]_{-2}^{0}+\frac{1}{2}\left[\frac{x^2}{2}\right]_{0}^{4/3}$$

$$=\frac{1}{8}\left[0-(4-\frac{32}{3}+8)\right]+\frac{1}{4}\left[\frac{16}{9}-0\right]$$

$$=-\frac{1}{6}+\frac{4}{9}$$

$$=\frac{5}{18}$$

Cumulative Distribution Function

For $(-\infty < x < -2)$

$$F(x) = P(-\infty < x < -2) = 0$$

For $(-2 \le x < 0)$

$$F(x) = F(-2) + \int_{-2}^{x} \frac{1}{8}(x+2)^2 \, dx$$

$$F(x) = 0 + \frac{1}{8}\left[\frac{(x+2)^3}{3}\right]_{-2}^{x}$$

$$F(x) = 0 + \frac{1}{8}\left[\frac{(x+2)^3}{3}\right]_{-2}^{x}$$

$$= \frac{1}{24}\left[(x+2)^3 - 0\right]$$

$$= \frac{1}{24}(x+2)^3$$

For $(0 \leq x \leq 1\frac{1}{3})$

$$F(x) = F(0) + \int_0^x \frac{1}{2}\,dx$$

For $(0 \leq x \leq 1\frac{1}{3})$

$$F(x) = F(0) + \int_0^x \frac{1}{2}\,dx$$

$$F(x) = \frac{1}{24}(0+2)^3 + \frac{1}{2}[x]_0^x = \frac{1}{3} + \frac{1}{2}[x-0] = \frac{1}{2}x + \frac{1}{3}$$

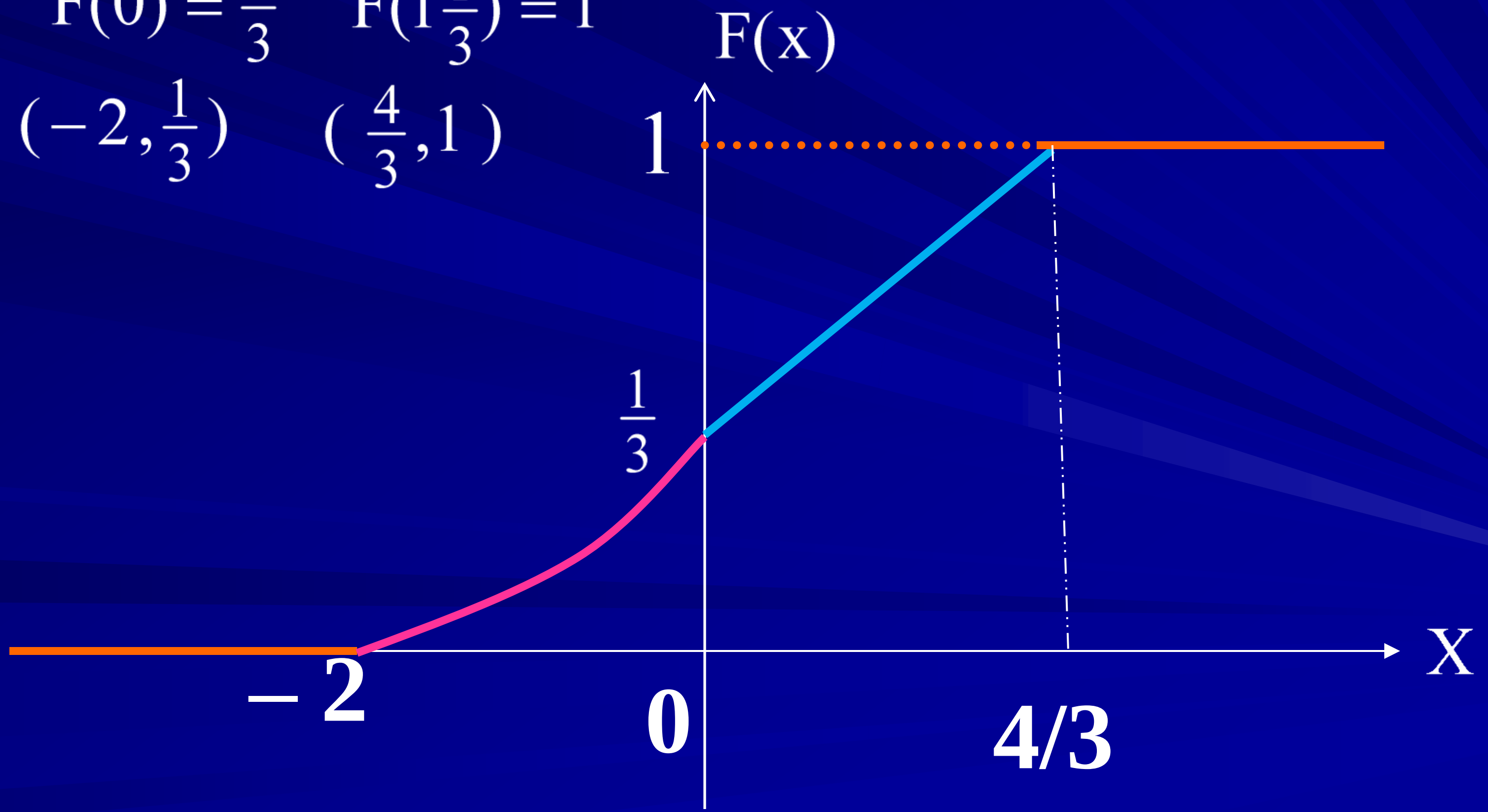
For $(1\frac{1}{3} < x < \infty)$

$$F(x) = F(1\frac{1}{3}) + 0 \quad = \frac{1}{2} \times \frac{4}{3} + \frac{1}{3} = 1$$

$$F(x)=\begin{cases}0 & ,-\infty< x<-2\\ \frac{1}{24}(x+2)^3 & ,\quad -2\le x<0\\ \frac{1}{2}x+\frac{1}{3} & ,\quad 0\le x\le 1\frac{1}{3}\\ 1 & ,\ 1\frac{1}{3}<x<\infty\end{cases}$$

$F(-2)=0$ $\quad F(0)=\frac{1}{3}$ $\quad F(1\frac{1}{3})=1$

$(-2,0)$ $\quad (-2,\frac{1}{3})$ $\quad (\frac{4}{3},1)$



Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{2} m + \frac{1}{3} = \frac{1}{2}$$

$$3m + 2 = 3$$

$$m = \frac{1}{3}$$

$$P(\ -1 \le x \le 1\ ) = F(1) - F(-1)$$

$$= \left(\frac{1}{2} \times 1 + \frac{1}{3}\right) - \frac{1}{24} \times (-1+2)^3$$

$$= \frac{20-1}{24}$$

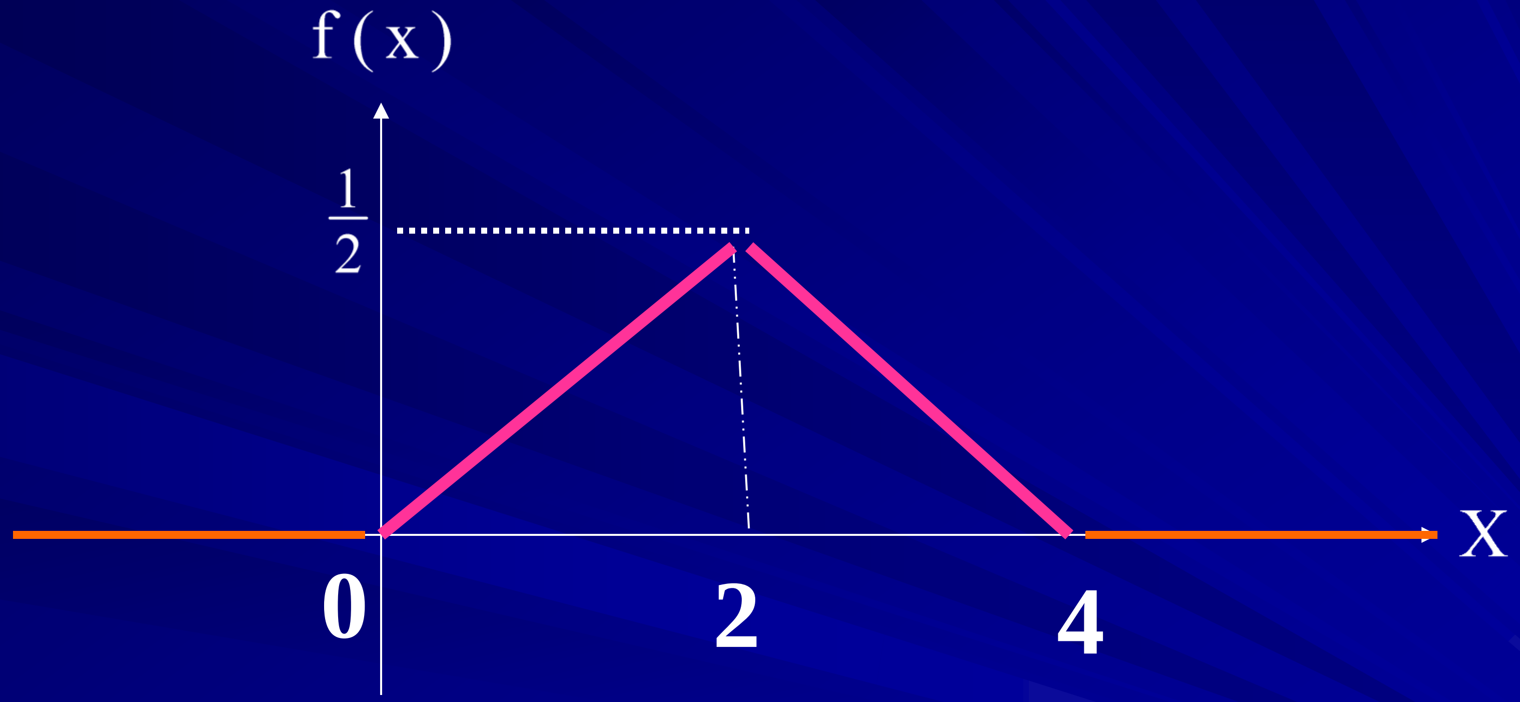
$$= \frac{19}{24}$$

$$f(x)=\begin{cases}\frac{1}{4}x & 0\le x<2\\ \frac{1}{4}(4-x) & 2\le x\le 4\\ 0 & otherwise\end{cases}$$

$$f(0)=0 \qquad f(2)=\frac{1}{2} \qquad f(2)=\frac{1}{2} \qquad f(4)=0$$

$$(0,0) \qquad (2,\frac{1}{2}) \qquad (2,\frac{1}{2}) \qquad (4,0)$$

$(0,0)$ $(2,\frac{1}{2})$ $(2,\frac{1}{2})$ $(4,0)$



$$E(X)=\int_0^2 \frac{1}{4}.x.x\,dx+\int_2^4 \frac{1}{4}(4-x)x\,dx$$

$$=\frac{1}{4}\int_0^2 x^2\,dx+\frac{1}{4}\int_2^4 (4x-x^2)\,dx$$

$$=\frac{1}{12}[x^3]_0^2+\frac{1}{4}[(2x^2-\frac{x^3}{3})]_2^4$$

$$=\frac{1}{12}[8-0]+\frac{1}{4}[(32-\frac{64}{3})-(8-\frac{8}{3})]$$

$$=\frac{2}{3}+8-\frac{16}{3}-2+\frac{2}{3}$$

$$=2$$

$$E(X^2) = \int_0^2 \frac{1}{4} . x . x^2 \, dx + \int_2^4 \frac{1}{4}(4 - x) x^2 \, dx$$

$$= \frac{1}{4} \int_0^2 x^3 \, dx + \frac{1}{4} \int_2^4 (4x^2 - x^3) \, dx$$

$$= \frac{1}{16} [x^4]_0^2 + \frac{1}{4} [(\frac{4x^3}{3} - \frac{x^4}{4})]_2^4$$

$$= \frac{1}{16} [16 - 0] + [(\frac{x^3}{3} - \frac{x^4}{16})]_2^4$$

$$= 1 + (\frac{64}{3} - 16) - (\frac{8}{3} - 1)$$

$$= 1 + \frac{64}{3} - 16 - \frac{8}{3} + 1 = \frac{56}{3} - 14 = \frac{56 - 42}{3}$$

$$= \frac{14}{3}$$

$$\mathrm{Var}(X) = E[X^2] - \{E[x]\}^2$$

$$= \frac{14}{3} - 2^2$$

$$= \frac{2}{3}$$

$$\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{\frac{2}{3}}$$

$$E(2X + 5) = 2E(X) + 5 = 2 \times 2 + 5 = 9$$

$$\mathrm{Var}(3X + 2) = 9\,\mathrm{Var}(X) = 9 \times \frac{2}{3} = 5$$

Cumulative Distribution Function

For $(-\infty < x < 0)$

$$F(x) = 0$$

For $(0 \le x < 2)$

$$F(x) = F(0) + \int_0^x f(x)\,dx$$

$$F(x) = 0 + \int_0^x \frac{1}{4}\,x\,dx = \frac{1}{8}\left[x^2\right]_0^x$$

$$= \frac{1}{8}\left[x^2 - 0\right] = \frac{1}{8}x^2$$

For $(2 \le x \le 4)$

$$F(x) = F(2) + \int_2^x \frac{1}{4}(4 - x)\,dx$$

$$F(x) = \frac{1}{8} \times 2^2 + \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_2^x = \frac{1}{2} + \left[x - \frac{1}{8}x^2\right]_2^x$$

$$= \frac{1}{2} + \left[(x - \frac{1}{8}x^2) - (2 - \frac{1}{2})\right]$$

$$= \frac{1}{2} + x - \frac{1}{8}x^2 - \frac{3}{2}$$

$$= -\frac{1}{8}x^2 + x - 1$$

For $(4 < x < \infty)$

$$F(x) = F(4) + 0$$

$$= -\frac{1}{8} \times 16 + 4 - 1$$

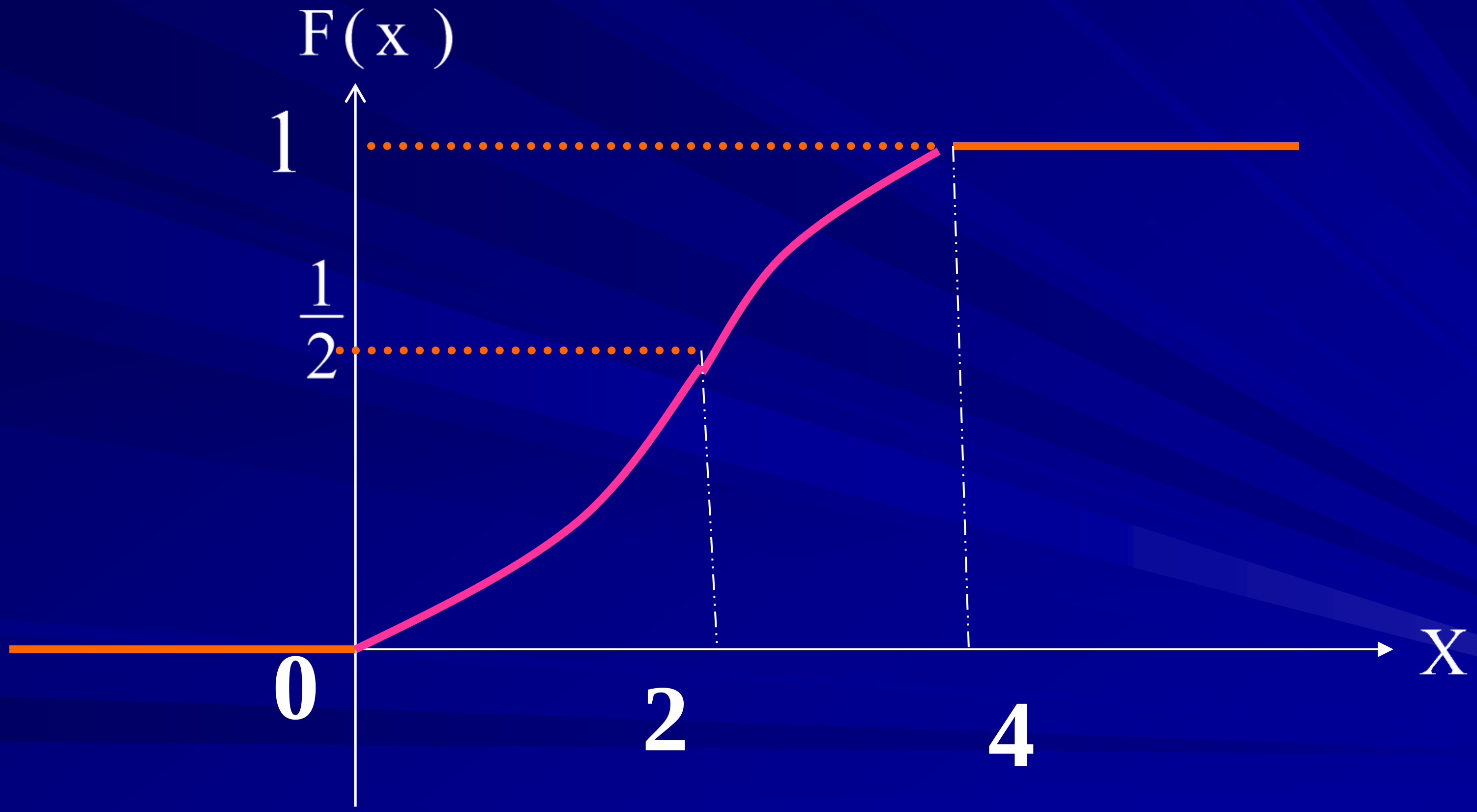
$$= 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{8}x^2 & , \quad 0 \le x < 2 \\ -\frac{1}{8}x^2 + x - 1 & , \ 2 \le x \le 4 \\ 1 & , \ 4 < x < \infty \end{cases}$$

$F(0) = 0$ $\quad F(2) = \frac{1}{2}$ $\quad F(4) = 1$

$(0, 0)$ $\quad (2, \frac{1}{2})$ $\quad (4, 1)$

$(0,0)$ $(2, \frac{1}{2})$ $(4,1)$



Median

$$F(m) = \frac{1}{2}$$

$$-\frac{1}{8}m^2 + m - 1 = \frac{1}{2}$$

$$m^2 - 8m + 12 = 0$$

$$(m-2)(m-6) = 0$$

$$m = 2 \text{ or } m = 6$$

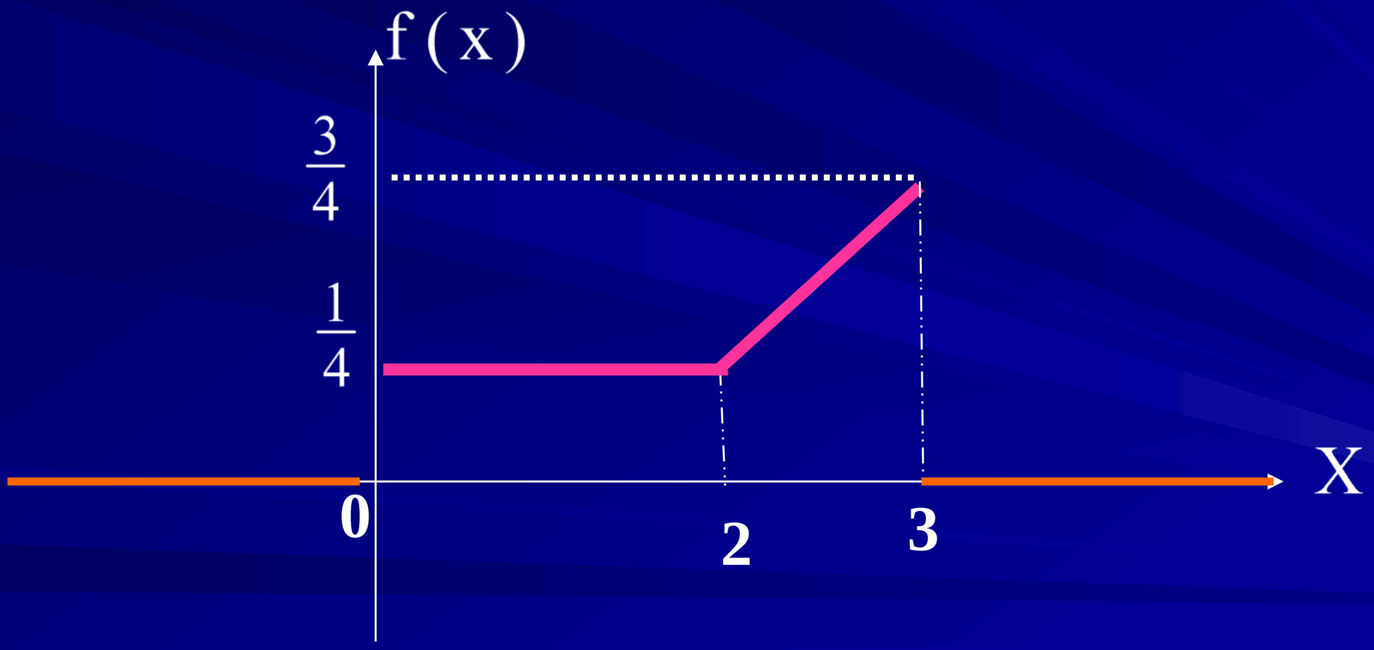
$$m = 6 \text{ impossible}$$

$$\therefore m = 2$$

$$f(x) = \begin{cases} \frac{1}{4} & , \ 0 \le x < 2 \\ \frac{1}{4}(2x-3) & , \ 2 \le x \le 3 \\ 0 & , \ \text{otherwise} \end{cases}$$

$$f(0) = \frac{1}{4} \qquad f(2) = \frac{1}{4} \qquad f(2) = \frac{1}{4} \qquad f(3) = \frac{3}{4}$$

$$\left(0, \frac{1}{4}\right) \qquad \left(2, \frac{1}{4}\right) \qquad \left(2, \frac{1}{4}\right) \qquad \left(3, \frac{3}{4}\right)$$



Cumulative Distribution Function

For $(-\infty < x < 0)$

$$F(x) = 0$$

For $(0 \le x < 2)$

$$F(x) = F(0) + \int_0^x \frac{1}{4}\,dx = 0 + \frac{1}{4}\Big[\;x\;\Big]_0^x = \frac{1}{4}(x-0) = \frac{1}{4}x$$

For $(2 \le x \le 3)$

$$F(x) = F(2) + \int_2^x \frac{1}{4}(2x-3)\,dx = \frac{1}{4}\times 2 + \frac{1}{4}\Big[x^2 - 3x\Big]_2^x$$

$$= \frac{1}{2} + \frac{1}{4}\Big[(x^2 - 3x) - (4-6)\Big]$$

$$= \frac{1}{2} + \frac{1}{4}\left[(x^2 - 3x) - (4 - 6)\right]$$

$$= \frac{1}{2} + \frac{1}{4}x^2 - \frac{3}{4}x + \frac{1}{2}$$

$$= \frac{1}{4}x^2 - \frac{3}{4}x + 1$$

For $(3 < x < \infty)$

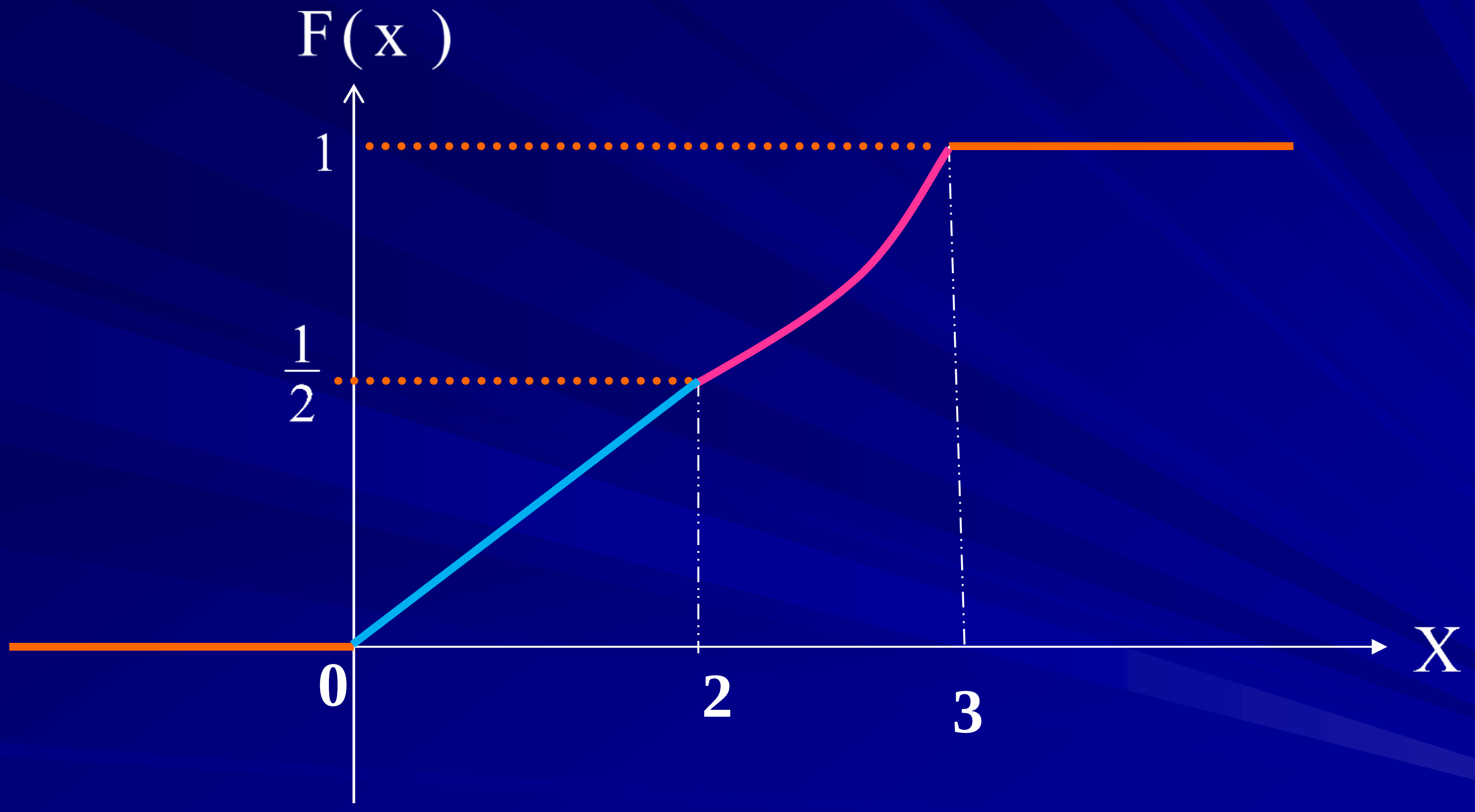
$$F(x) = F(3) + 0 = \frac{1}{4}\times 3^2 - \frac{3}{4}\times 3 + 1 = 1$$

$$F(x)=\begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{4}x & , 0 \le x < 2 \\ \frac{1}{4}x^2 - \frac{3}{4}x + 1 & , 2 \le x \le 3 \\ 1 & , 3 < x < \infty \end{cases}$$

$F(0)=0$ $\quad$ $F(2)=\frac{1}{2}$ $\quad$ $F(3)=1$

$(0, 0)$ $\quad$ $(2, \frac{1}{2})$ $\quad$ $(3, 1)$

$(0, 0)$ $(2, \frac{1}{2})$ $(3, 1)$



If we calculate the median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{4}m^2 - \frac{3}{4}m + 1 = \frac{1}{2}$$

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2) = 0$$

$$m = 1 \text{ or } m = 2$$

Moreover

$$F(m) = \frac{1}{2}$$

$$\frac{1}{4}m = \frac{1}{2}$$

$$m = 2$$

Since, median is existence and uniqueness

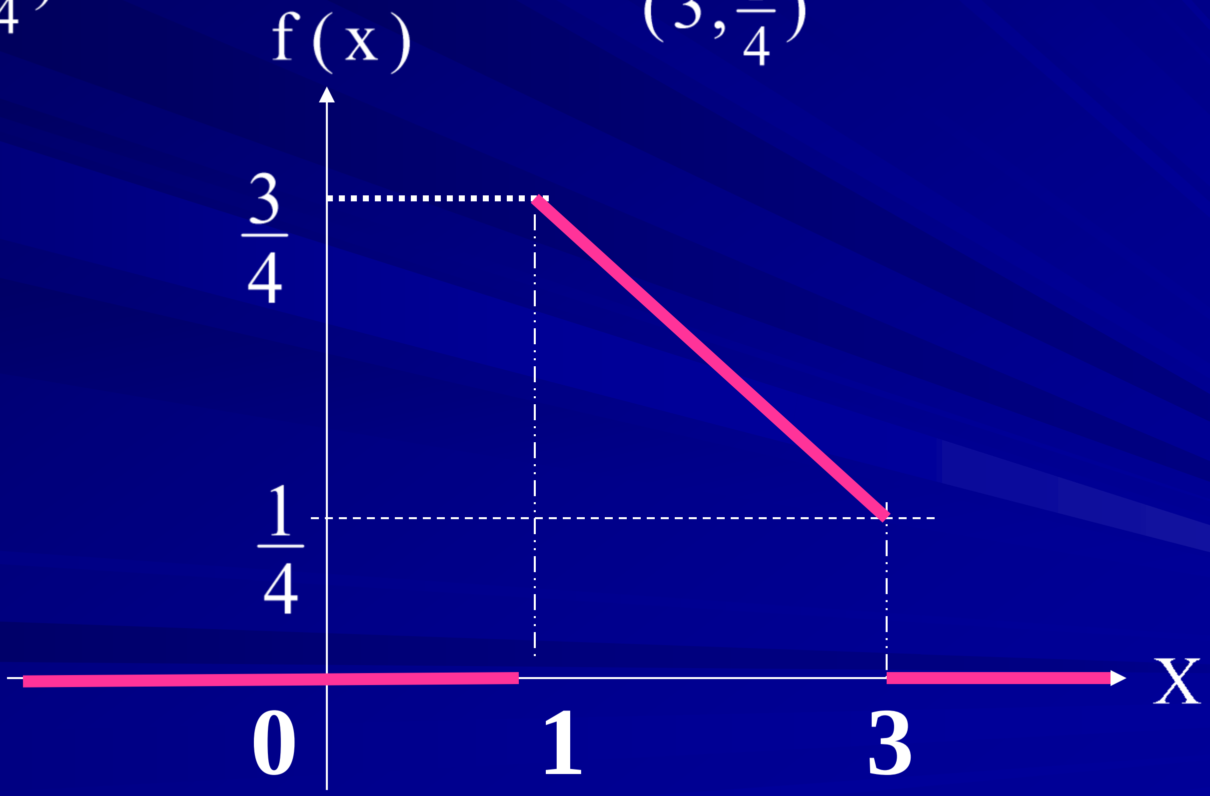
$m = 1$ is impossible

$$\therefore m = 2$$

$$f(x)=\begin{cases}\frac{1}{4}(4-x), & 1\le x\le 3\\ 0 & otherwise\end{cases}$$

$$f(1)=\frac{3}{4} \qquad f(3)=\frac{1}{4}$$

$$(1,\tfrac{3}{4}) \qquad (3,\tfrac{1}{4})$$



$$E(X)=\int_{1}^{3} \frac{1}{4}(4-x)x \, dx$$

$$=\frac{1}{4}\int_{1}^{3} (4x-x^2)\, dx$$

$$=\frac{1}{4}\left[2x^2-\frac{1}{3}x^3\right]_1^3$$

$$=\frac{1}{4}\left[(18-9)-\left(2-\frac{1}{3}\right)\right]$$

$$=\frac{11}{6}$$

$$E(X^2) = \int_1^3 \frac{1}{4}(4 - x)\,x^2 \; dx$$

$$E(X^2) = \frac{1}{4}\int_1^3 (4x^2 - x^3)\; dx$$

$$E(X^2) = \frac{1}{4}\left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_1^3$$

$$E(X^2) = \frac{1}{4}\left[\left(36 - \frac{81}{4}\right) - \left(\frac{4}{3} - \frac{1}{4}\right)\right]$$

$$E(X^2) = \frac{1}{4}\left[36 - \frac{81}{4} - \frac{4}{3} + \frac{1}{4}\right)] \quad = \frac{11}{3}$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{11}{3} - \left(\frac{11}{6}\right)^2 = \frac{11}{36}$$

$$\sigma = \frac{\sqrt{11}}{6}$$

Median

$$P(1 \le x \le m) = \int_1^m \frac{1}{4}(4 - x)\,dx = \frac{1}{2}$$

$$-\frac{1}{8}\left[(4 - x)^2\right]_1^m = \frac{1}{2}$$

$$(4 - m)^2 - (4 - 1)^2 = -4$$

$$(4 - m)^2 = 5$$

$$m = 4 - \sqrt{5} \quad \text{or} \quad m = 4 + \sqrt{5} \ (\text{impossible})$$

$$\therefore m = 4 - \sqrt{5}$$

Cumulative Distribution Function

For $-\infty < x < 1$

$$F(x) = 0$$

For $1 \le x \le 3$

$$F(x) = F(1) + \int_{1}^{x} \frac{1}{4}(4 - x)\,dx$$

$$= 0 - \frac{1}{8}\left[(4 - x)^2\right]_{1}^{x}$$

$$= -\frac{1}{8}\left[(4 - x)^2 - (4 - 1)^2\right]$$

$$= -\frac{1}{8}x^2 - x + \frac{7}{8}$$

Cumulative Distribution Function

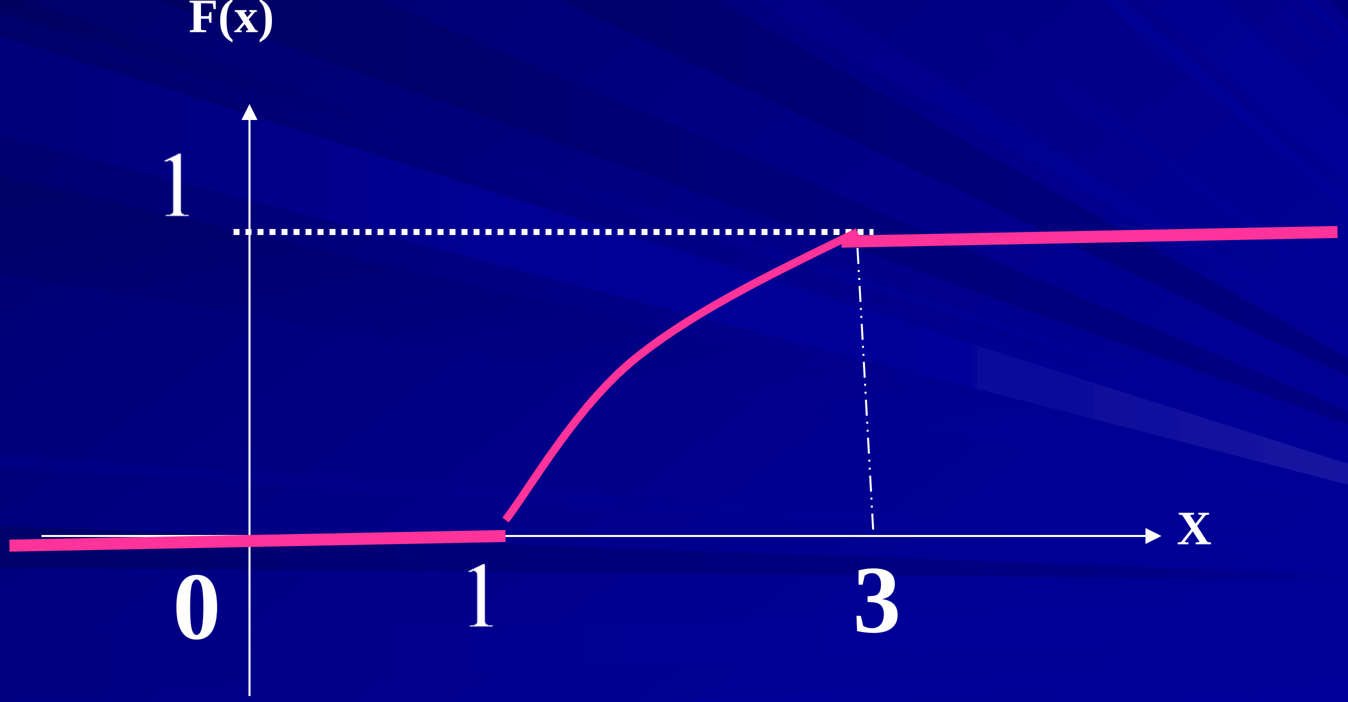
For $3 < x < \infty$

$$F(x) = F(3) + 0 = \frac{1}{8} \times 3^2 - 3 + \frac{7}{8} = 1$$

$$f(x) = \begin{cases} 0 & , \quad -\infty < x < 1 \\ -\frac{1}{8}x^2 - x + \frac{7}{8} & , 1 \le x \le 3 \\ 1 & , \; 3 < x < \infty \end{cases}$$

$F(1) = 0$ $F(3) = 1$

$(1,0)$ $(3,1)$



Median

$$F(m) = \frac{1}{2}$$

$$-\frac{1}{8}m^2 - m + \frac{7}{8} = \frac{1}{2}$$

$$m^2 - 8m + 7 = -4$$

$$m^2 - 8m + 16 = 5$$

$$(m - 4)^2 = 5$$

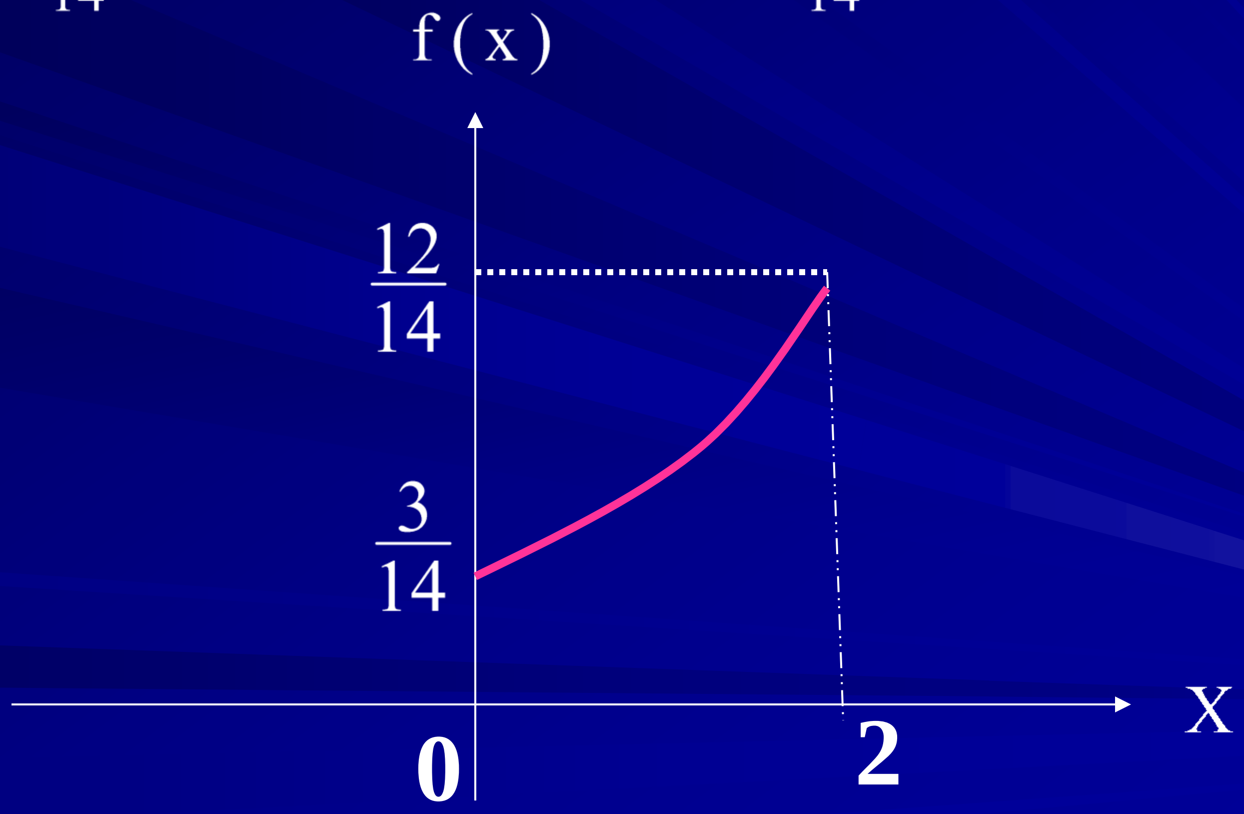
$$m = 4 - \sqrt{5} \quad \text{or} \quad m = 4 + \sqrt{5} \ (\text{impossible})$$

$$m = 4 - \sqrt{5}$$

$$f(x) = \frac{3}{56}(x+2)^2 \quad , 0 \le x \le 2$$

$$f(0) = \frac{3}{14} \qquad f(2) = \frac{12}{14}$$

$$(0, \frac{3}{14}) \qquad (2, \frac{12}{14})$$



$$P(0 \le x \le 1) = \int_0^1 \frac{3}{56}(x+2)^2\, dx$$

$$= \frac{1}{56}\left[(x+2)^3\right]_0^1$$

$$= \frac{1}{56}[27 - 8]$$

$$= \frac{19}{56}$$

$$P(x \geq 1) = \int_{1}^{2} \frac{3}{56}(x+2)^2 \, dx$$

$$= \frac{1}{56}\left[(x+2)^3\right]_1^2$$

$$= \frac{1}{56}\left[64 - 27\right]$$

$$= \frac{37}{56}$$

$$E(X)=\int_0^2 \frac{3}{56}(x+2)^2\,x\,dx = \frac{3}{56}\int_0^2 (x^3+4x^2+4x)\,dx$$

$$=\frac{3}{56}\left[\frac{x^4}{4}+\frac{4x^3}{3}+2x^2\right]_0^2 = \frac{3}{56}\left[(4+\frac{32}{3}+8)-0\right]$$

$$=\frac{3}{14}\left[1+\frac{8}{3}+2\right] = \frac{3}{14}\times\frac{17}{3} = \frac{17}{14}$$

$$E(X^2)=\int_0^2 \frac{3}{56}(x+2)^2\,x^2\,dx = \frac{3}{56}\int_0^2 (x^4+4x^3+4x^2)\,dx$$

$$=\frac{3}{56}\left[\frac{x^5}{5}+x^4+\frac{4x^3}{3}\right]_0^2 = \frac{3}{56}\left[(\frac{32}{5}+16+\frac{32}{3})-0\right]$$

$$=\frac{12}{35}+\frac{30}{35}+\frac{20}{35}=\frac{62}{35}$$

Cumulative Distribution Function

For $0 \le x \le 2$

$$F(x) = \int_0^x \frac{3}{56}(x+2)^2\,dx$$

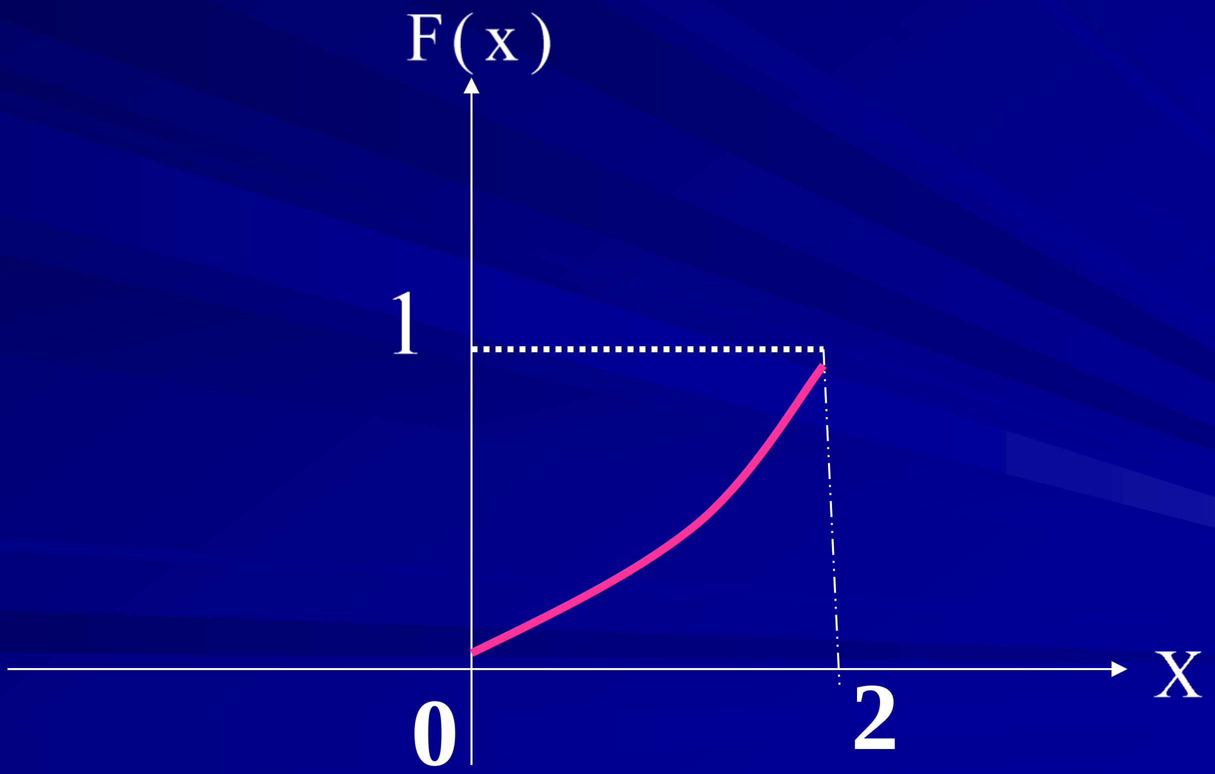
$$= \frac{1}{56}\left[(x+2)^3\right]_0^x$$

$$= \frac{1}{56}\left[(x+2)^3 - 8\right]$$

$$F(x) = \frac{1}{56}[(x+2)^3 - 8] \quad , 0 \le x \le 2$$

$F(0) = 0$ $F(2) = 1$

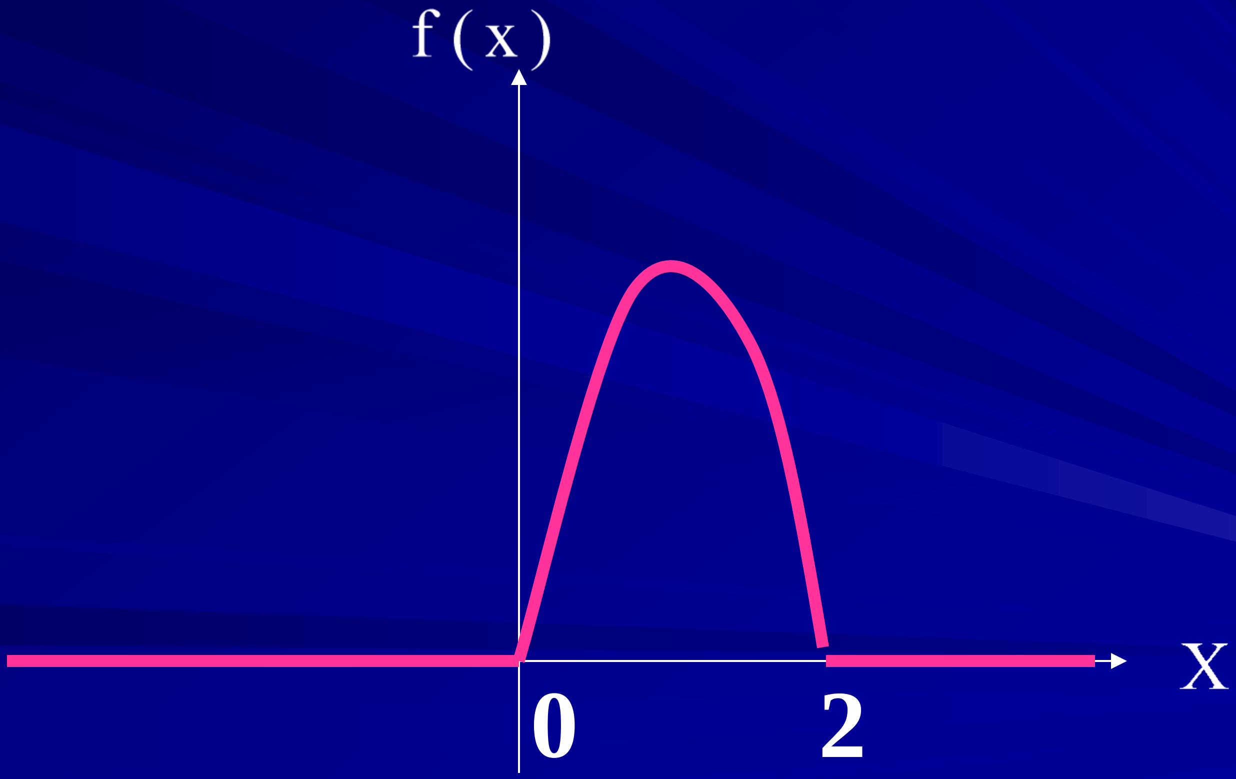
(0,0) (2,1)



$$f(x)=\begin{cases}\frac{3}{4}x(2-x)), & 0\le x\le 2\\ 0 & otherwise\end{cases}$$

$f(1)=0$ $f(2)=0$

$(1,0)$ $(2,0)$



$$E(X)=\int_0^2 \frac{3}{4}x(2-x)x\;dx=\frac{3}{4}\int_0^2 (2x^2-x^3)\;dx$$

$$=\frac{3}{4}\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_0^2=\frac{3}{4}\left[\left(\frac{16}{3}-4\right)-0\right]=\frac{3}{4}\times\frac{4}{3}=1$$

$$E(X^2)=\int_0^2 \frac{3}{4}x(2-x)x^2\;dx=\frac{3}{4}\int_0^2 (2x^3-x^4)\;dx$$

$$=\frac{3}{4}\left[\frac{x^4}{2}-\frac{x^5}{5}\right]_0^2=\frac{3}{4}\left[\left(8-\frac{32}{5}\right)-0\right]=\frac{3}{4}\times\frac{8}{5}=\frac{6}{5}$$

$$Var(X)=E(X^2)-[E(X)]^2=\frac{6}{5}-1^2=\frac{1}{5}$$

$$\sigma=\sqrt{\frac{1}{5}}$$

For $(-\infty < x < 0)$

$$F(x) = P(-\infty < x < x) = 0$$

For $(0 \le x \le 2)$

$$F(x) = F(0) + \int_0^x \frac{3}{4} x(2-x)\,dx = 0 + \frac{3}{4}\int_0^x (2x - x^2)\,dx$$

$$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^x = \frac{3}{4}\left[\left(x^2 - \frac{x^3}{3}\right) - 0\right] = -\frac{1}{4}x^3 + \frac{3}{4}x^2$$

For $(2 < x < \infty)$

$$F(x) = F(2) + 0 = -\frac{1}{4}\times 2^3 + \frac{3}{4}\times 2^2 = 1$$

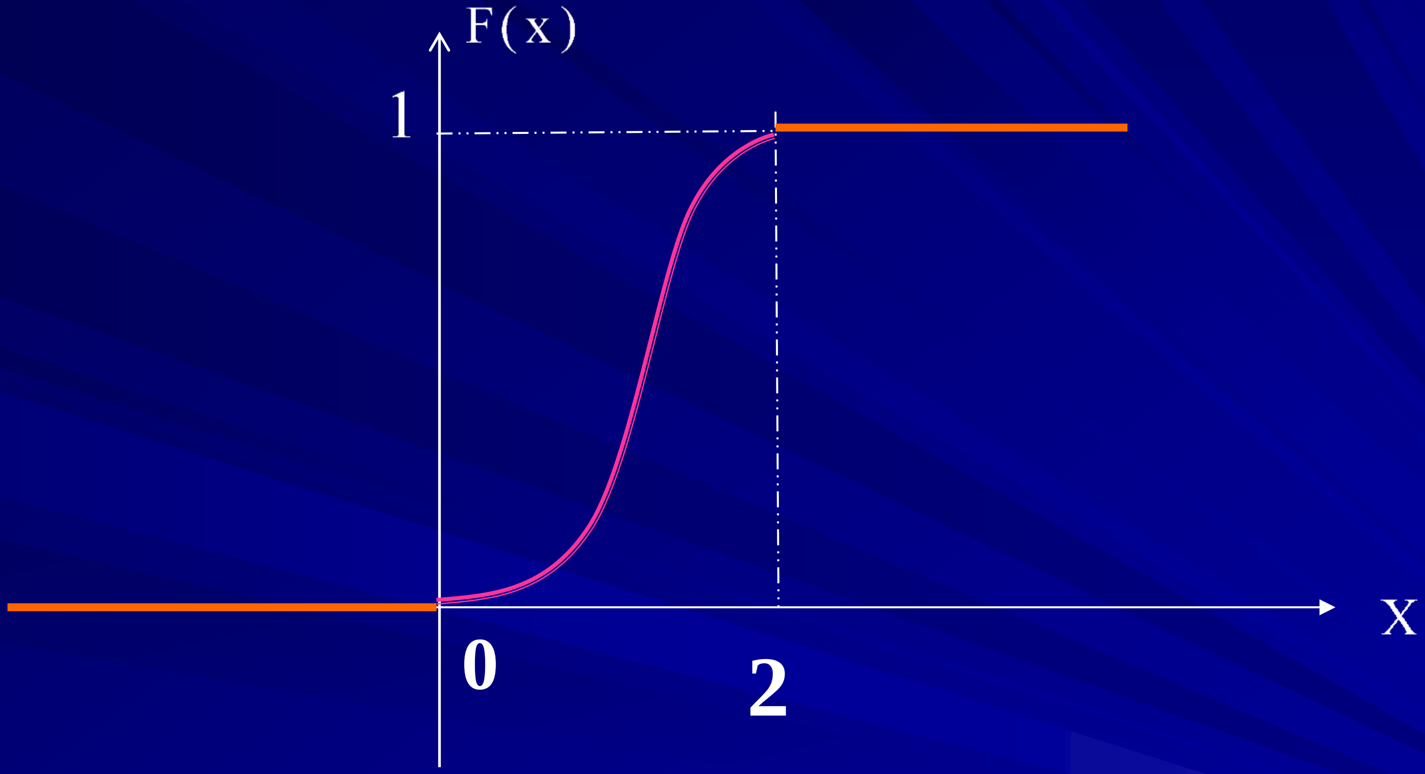
$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ -\frac{1}{4}x^3 + \frac{3}{4}x^2 & , \quad 0 \le x \le 2 \\ 1 & , \ 2 < x < \infty \end{cases}$$

$F(0)=0$

$(0,0)$

$F(2)=1$

$(2,1)$



$$F(m)=\frac{1}{2}$$

$$F(m) = \frac{1}{2}$$

$$-\frac{1}{4}m^3 + \frac{3}{4}m^2 = \frac{1}{2}$$

$$m^3 - 3m^2 + 2 = 0$$

$$F(m) = m^3 - 3m^2 + 2$$

$$F(1) = 1 - 3 + 2 = 0$$

Since, median is existence and uniqueness

$$\therefore m = 1$$

A continuous random variable X has probability density function f (x) where

$$f(x) = \begin{cases} \frac{6}{7}x & , 0 \le x < 1 \\ \frac{6}{7}x(2-x) & , 1 \le x \le 2 \\ 0 & , \text{otherwise} \end{cases}$$

(a)find E (X)

(b)find $E(X^2)$

$$E(X)=\int_0^1 \frac{6}{7}.x.x\,dx+\int_1^2 \frac{6}{7}x(2-x)x\,dx$$

$$=\frac{6}{7}\int_0^1 x^2\,dx+\frac{6}{7}\int_1^2 (2x^2-x^3)\,dx$$

$$=\frac{2}{7}\,[x^3]_0^1+\frac{6}{7}\,[(\frac{2}{3}x^3-\frac{x^4}{4})]_1^2$$

$$=\frac{2}{7}\,[1-0]+\frac{6}{7}\,[(\frac{16}{3}-4)-(\frac{2}{3}-\frac{1}{4})]$$

$$=\frac{2}{7}+\frac{6}{7}\,[\frac{4}{3}-\frac{2}{3}+\frac{1}{4}]$$

$$=\frac{2}{7}+\frac{6}{7}\times\frac{11}{12}$$

$$=\frac{15}{14}$$

$$E(X^2) = \int_0^1 \frac{6}{7} . x . x^2 \, dx + \int_1^2 \frac{6}{7} x(2-x)x^2 \, dx$$

$$= \frac{6}{7} \int_0^1 x^3 \, dx + \frac{6}{7} \int_1^2 (2x^3 - x^4) \, dx$$

$$= \frac{3}{14} [x^4]_0^1 + \frac{6}{7} [(\frac{1}{2}x^4 - \frac{x^5}{5})]_1^2$$

$$= \frac{3}{14} [1 - 0] + \frac{6}{7} [(8 - \frac{32}{5}) - (\frac{1}{2} - \frac{1}{5})]$$

$$= \frac{3}{14} + \frac{6}{7} [\frac{8}{5} - \frac{3}{10}] \quad = \frac{3}{14} + \frac{6}{7} \times \frac{13}{10}$$

$$= \frac{3}{14} + \frac{6}{7} \times \frac{13}{10} \quad = \frac{3}{14} + \frac{39}{35}$$

$$= \frac{93}{70}$$

Cumulative Distribution Function

For $-\infty < x < 0$

$$F(x) = 0$$

For $0 \le x < 1$

$$F(x) = F(0) + \int_0^x \frac{6}{7} x \, dx = 0 + \frac{6}{7}\left[\frac{x^2}{2}\right]_0^x = \frac{3}{7} x^2$$

For $1 \le x \le 2$

$$F(x) = F(1) + \int_1^x \frac{6}{7}(2x - x^2)\, dx = \frac{3}{7} \times 1^2 + \frac{6}{7}\left[x^2 - \frac{x^3}{3}\right]_1^x$$

$$= \frac{3}{7} + \frac{6}{7}\left[(x^2 - \frac{x^3}{3}) - (1 - \frac{1}{3})\right]$$

$$= \frac{3}{7} + \frac{6}{7}x^2 - \frac{2}{7}x^3 - \frac{4}{7}$$

$$= -\frac{2}{7}x^3 + \frac{6}{7}x^2 - \frac{1}{7}$$

For $2 < x < \infty$

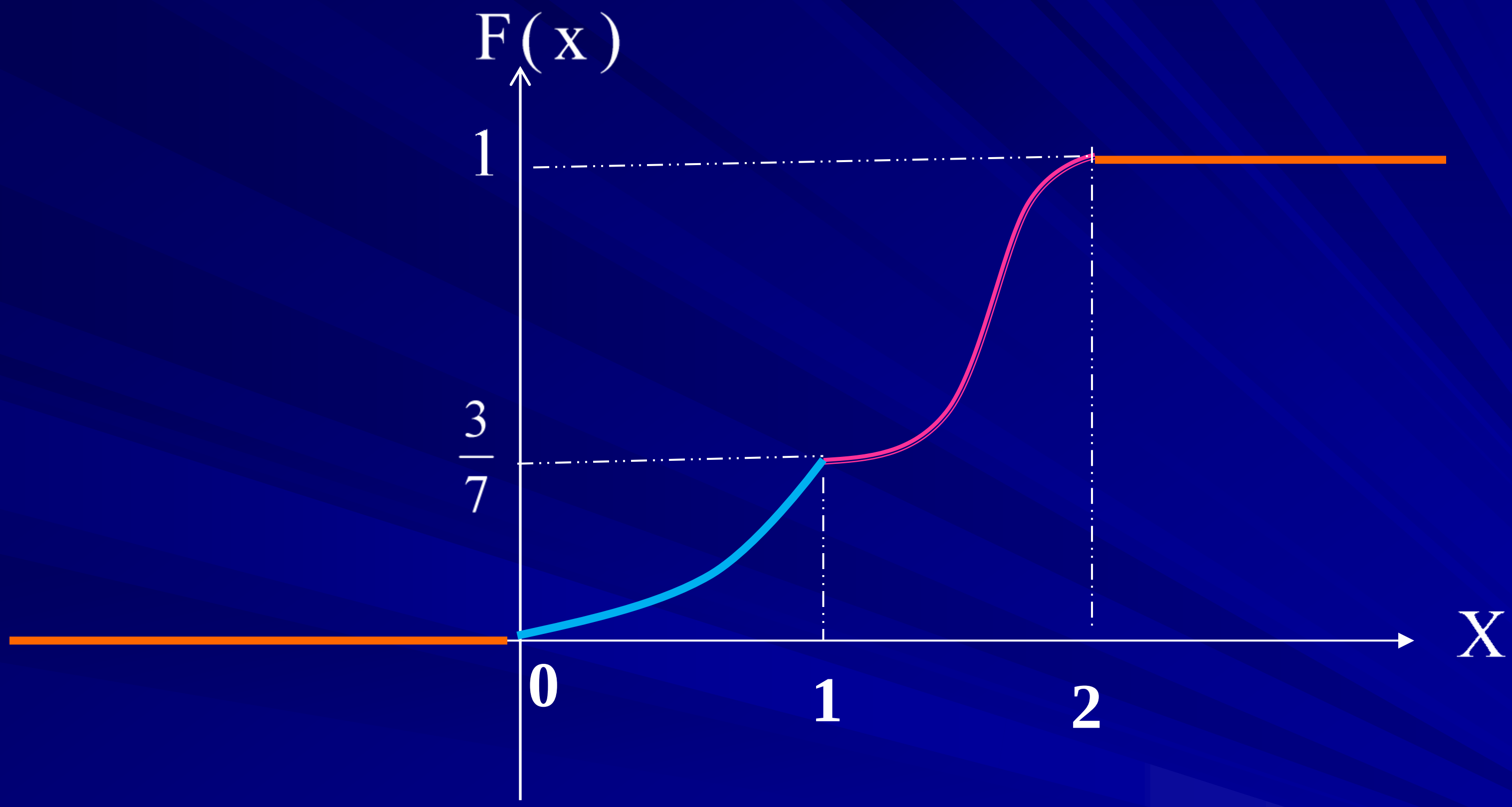
$$F(x) = F(2) + 0 = -\frac{2}{7} \times 2^3 + \frac{6}{7} \times 2^2 - \frac{1}{7} = \frac{-16 + 24 - 1}{7} = 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{3}{7}x^2 & , \quad 0 \le x < 1 \\ -\frac{2}{7}x^3 + \frac{6}{7}x^2 - \frac{1}{7} & , \ 1 \le x \le 2 \\ 1 & , \ 2 < x < \infty \end{cases}$$

$F(0) = 0$ $\quad$ $F(1) = \frac{3}{7}$ $\quad$ $F(2) = 1$

$(0, 0)$ $\quad$ $(1, \frac{3}{7})$ $\quad$ $(2, 1)$

$(0,0)$ $\quad (1,\frac{3}{7})$ $\quad (1,\frac{3}{7})$



A continuous random variable X has probability density function f (x) where

$$f(x) = \begin{cases} \frac{x}{3} & , \quad 0 \le x < 2 \\ -\frac{2x}{3} + 2 & , \quad 2 \le x \le 3 \\ 0 & , \text{ otherwise} \end{cases}$$

(a)sketch y = f (x) (b)sketch y = F (x)

(c)find $P(\ 1 \le x \le 2.5)$

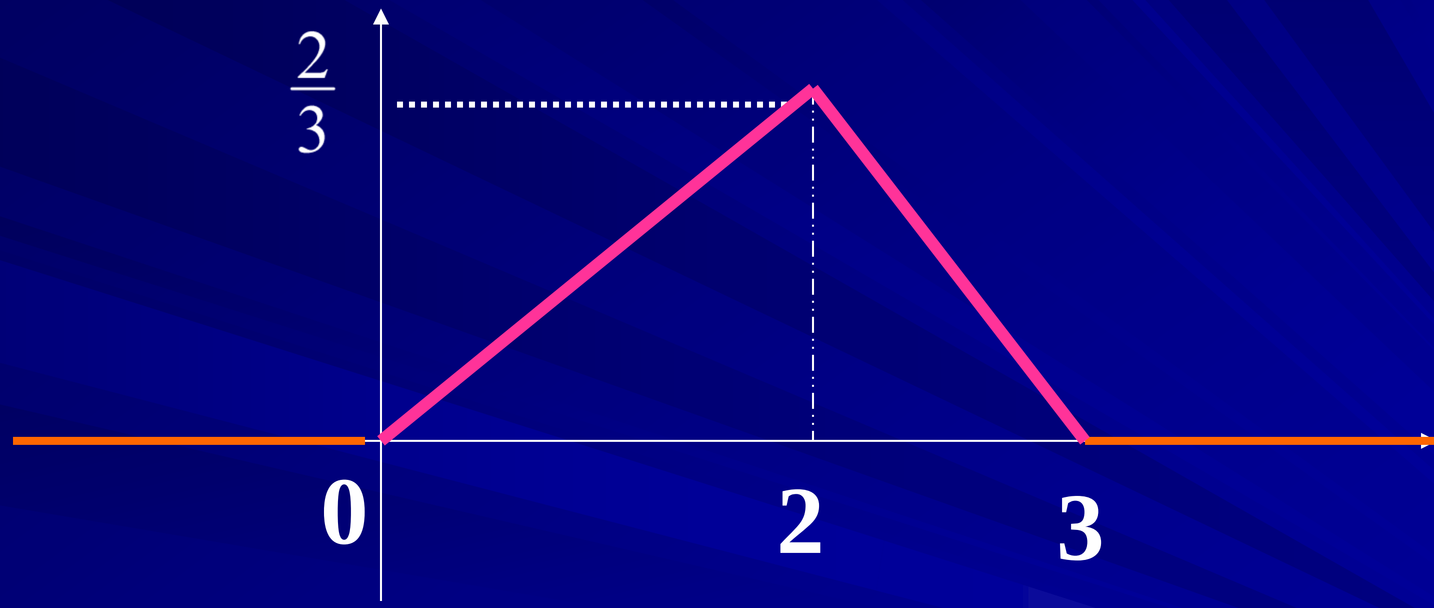
(d)find median

$$f(x) = \begin{cases} \frac{1}{3}x & , \quad 0 \le x < 2 \\ -\frac{2}{3}x + 2 & , \quad 2 \le x \le 3 \\ 0 & , \text{ otherwise} \end{cases}$$

$$f(0) = 0 \qquad f(2) = \frac{2}{3} \qquad f(3) = 0$$

$$(0,0) \qquad (2, \tfrac{2}{3}) \qquad (3,0)$$

$(0,0)$ $\left(2,\frac{2}{3}\right)$ $(3,0)$



Cumulative Distribution Function

$$For\ (-\infty < x < 0)$$

$$F(x) = P(-\infty < x < x) = 0$$

$$For\ (0 \le x < 2)$$

$$F(x) = F(0) + \int_0^x f(x)\,dx$$

$$F(x) = 0 + \int_0^x \frac{1}{3}\,x\,dx = \frac{1}{6}\left[x^2\right]_0^x = \frac{1}{6}x^2$$

For $(2 \leq x \leq 3)$

$$F(x) = F(2) + \int_{2}^{x} \left(-\frac{2}{3}x + 2\right)dx$$

$$F(x) = \frac{2}{3} + \left[-\frac{1}{3}x^2 + 2x\right]_2^x$$

$$= \frac{2}{3} + \left[\left(-\frac{1}{3}x^2 + 2x\right) - \left(-\frac{4}{3} + 4\right)\right]$$

$$= \frac{2}{3} - \frac{1}{3}x^2 + 2x + \frac{4}{3} - 4$$

$$= -\frac{1}{3}x^2 + 2x - 2$$

$$For\,(3 < x < \infty)$$

$$F(x) = F(3) + 0$$

$$= -\frac{1}{3} \times 3^2 + 2 \times 3 - 2$$

$$= 1$$

$$F(x)=\begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{6}x^2 & , 0 \le x < 2 \\ -\frac{1}{3}x^2+2x-2 & , 2 \le x \le 3 \\ 1 & , 3 < x < \infty \end{cases}$$

$F(0)=0$ $F(2)=\frac{2}{3}$ $F(3)=1$

$(0, 0)$ $(2, \frac{2}{3})$ $(3, 1)$

$F(0) = 0$

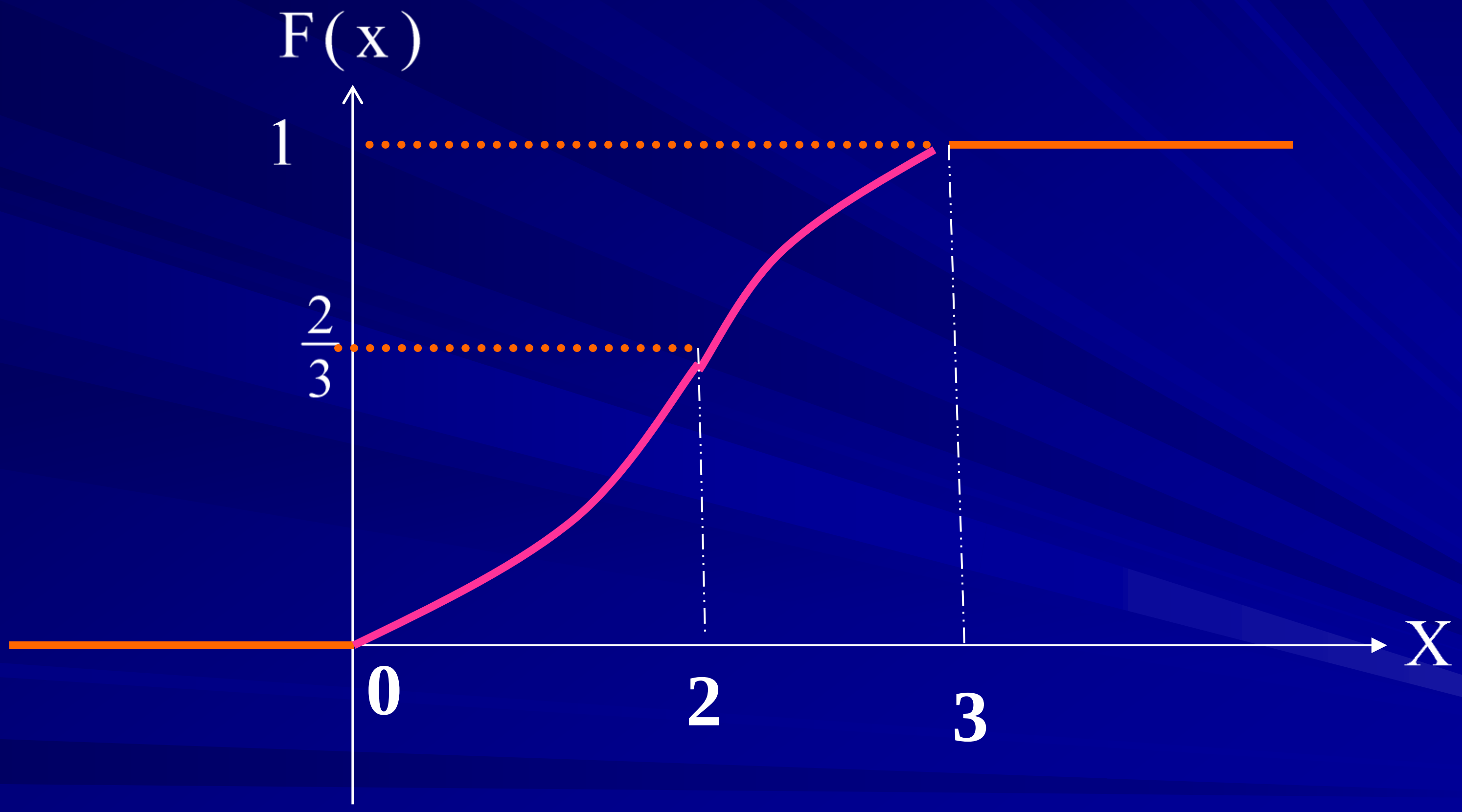
$(0, 0)$

$F(2) = \frac{2}{3}$

$(2, \frac{2}{3})$

$F(3) = 1$

$(3, 1)$



Median

$$P(0 \le x \le m) = \int_0^m \frac{1}{3} x \, dx = \frac{1}{2}$$

$$\frac{1}{6}[x^2]_0^m = \frac{1}{2}$$

$$m^2 = 3$$

$$m = \pm \sqrt{3}$$

$$\therefore m = \sqrt{3}$$

Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{6}m^2 = \frac{1}{2}$$

$$m^2 = 3$$

$$m = \pm\sqrt{3}$$

$$\therefore m = \sqrt{3}$$

$$P(1 \le x \le 2\frac{1}{2}) = \int_{1}^{2} \frac{1}{3} x \, dx + \int_{2}^{\frac{5}{2}} (-\frac{2}{3} x + 2) \, dx$$

$$= \frac{1}{6}\left[x^2\right]_1^2 + \left[-\frac{1}{3}x^2 + 2x\right]_2^{\frac{5}{2}}$$

$$= \frac{1}{6}\left[4 - 1\right] + \left[(-\frac{25}{12} + 5) - (-\frac{4}{3} + 4)\right]$$

$$= \frac{1}{2} + \left[-\frac{25}{12} + 5 + \frac{4}{3} - 4\right]$$

$$= \frac{1}{2} - \frac{3}{4} + 1 = \frac{3}{4}$$

A continuous random variable X has probability density function f (x) where

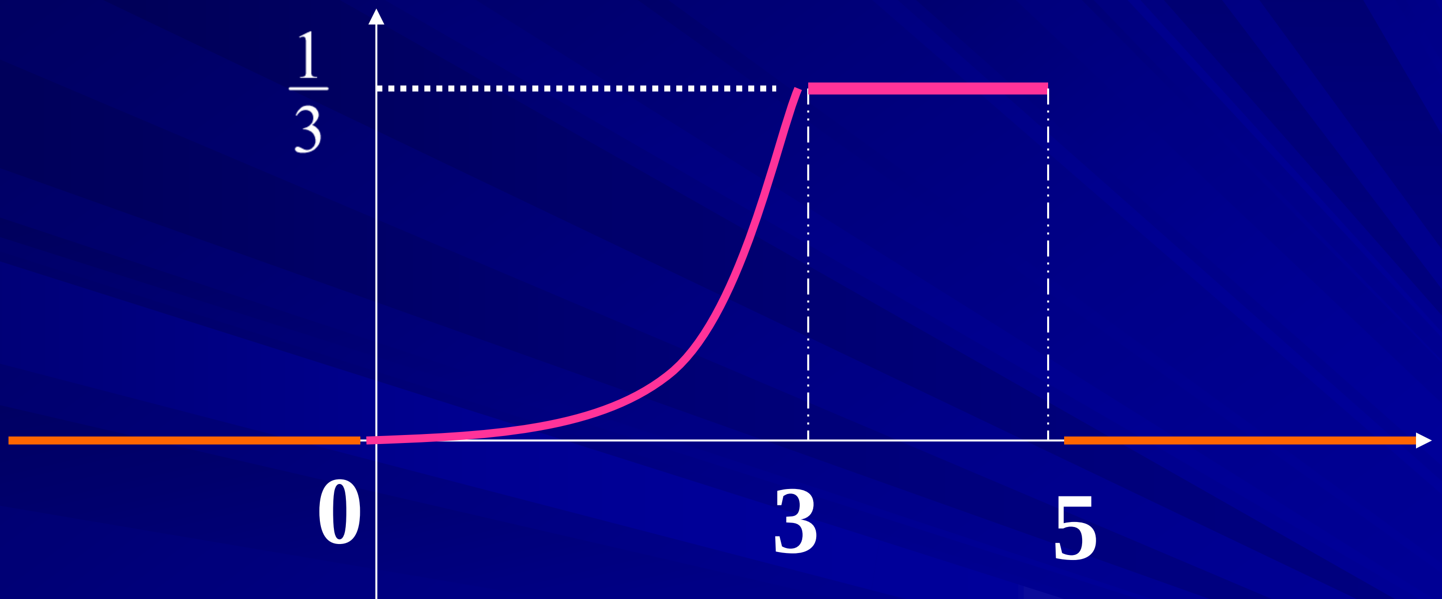
$$f(x) = \begin{cases} \frac{1}{27}x^2 & , \quad 0 \le x < 3 \\ \frac{1}{3} & , \quad 3 \le x \le 5 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Sketch y = f (x) (b) Find $E(X)$

(c) Find $E(X^2)$

(d) Find standard deviation

$(0,0)$ $\qquad$ $(3,\frac{1}{3})$ $\qquad$ $(5,\frac{1}{3})$



$$E(X)=\int_0^3 \frac{1}{27}.x^2.x\,dx+\int_3^5 \frac{1}{3}.x\,dx$$

$$E(X)=\int_0^3 \frac{1}{27}.x^3\,dx+\int_3^5 \frac{1}{3}\,x\,dx$$

$$E(X)=\frac{1}{108}[x^4]_0^3+\frac{1}{6}[x^2]_3^5$$

$$E(X)=\frac{1}{108}[3^4-0]+\frac{1}{6}[25-9]$$

$$E(X)=\frac{3}{4}+\frac{8}{3}$$

$$E(X)=\frac{41}{12}$$

$$E(X^2)=\int_0^3 \frac{1}{27}.x^2.x^2\,dx+\int_3^5 \frac{1}{3}.x^2\,dx$$

$$E(X^2)=\int_0^3 \frac{1}{27}.x^4\,dx+\int_3^5 \frac{1}{3}x^2\,dx$$

$$E(X^2)=\frac{1}{27\times 5}[x^5]_0^3+\frac{1}{9}[x^3]_3^5$$

$$E(X^2)=\frac{1}{27\times 5}[3^5-0]+\frac{1}{9}[125-27]$$

$$E(X^2)=\frac{9}{5}+\frac{98}{9}$$

$$E(X^2)=\frac{81+490}{45}=\frac{571}{45}$$

Cumulative Distribution Function

For $-\infty < x < 0$

$$F(x) = 0$$

For $0 \le x < 3$

$$F(x) = F(0) + \int_0^x \frac{1}{27} x^2 \, dx = 0 + \frac{1}{27}\left[\frac{x^3}{3}\right]_0^x = \frac{1}{81} x^3$$

For $3 \le x \le 5$

$$F(x) = F(3) + \int_3^x \frac{1}{3} \, dx = \frac{1}{81} \times 3^3 + \frac{1}{3}[x]_3^x = \frac{1}{3} + \frac{1}{3} x - 1 = \frac{1}{3} x - \frac{2}{3}$$

For $5 < x < \infty$

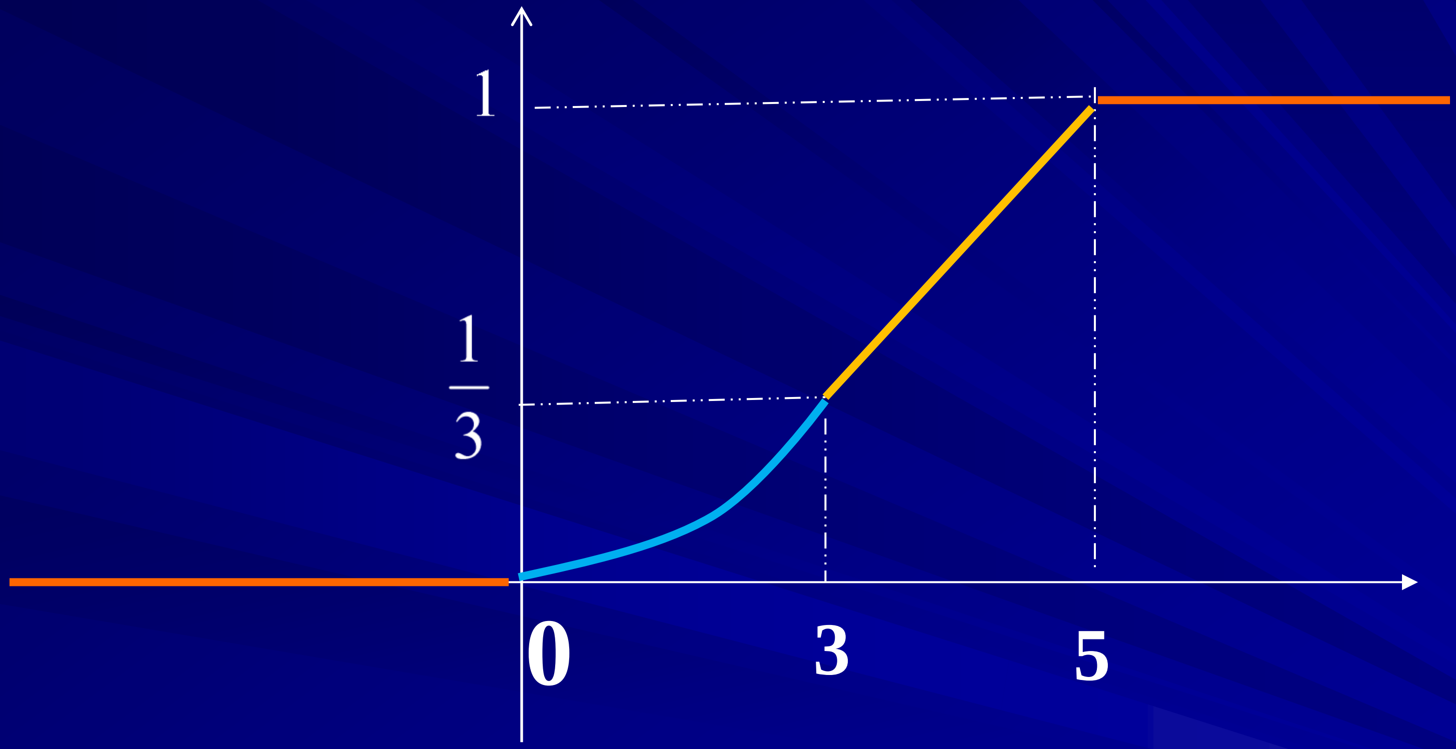
$$F(x) = F(5) + 0 = \frac{1}{3} \times 5 - \frac{2}{3} = 1$$

$$F(x) = \begin{cases} 0 & , -\infty < x < 0 \\ \frac{1}{81}x^3 & , \quad 0 \le x < 3 \\ \frac{1}{3}x - \frac{2}{3} & , \ 3 \le x \le 5 \\ 1 & , \ 5 < x < \infty \end{cases}$$

$F(0) = 0$ $\quad$ $F(3) = \frac{1}{3}$ $\quad$ $F(5) = 1$

$(0, 0)$ $\quad$ $(3, \frac{1}{3})$ $\quad$ $(5, 1)$

$(0, 0)$ $(3, \frac{1}{3})$ $(5, 1)$



Median

$$F(m) = \frac{1}{2}$$

$$\frac{1}{3}m - \frac{2}{3} = \frac{1}{2}$$

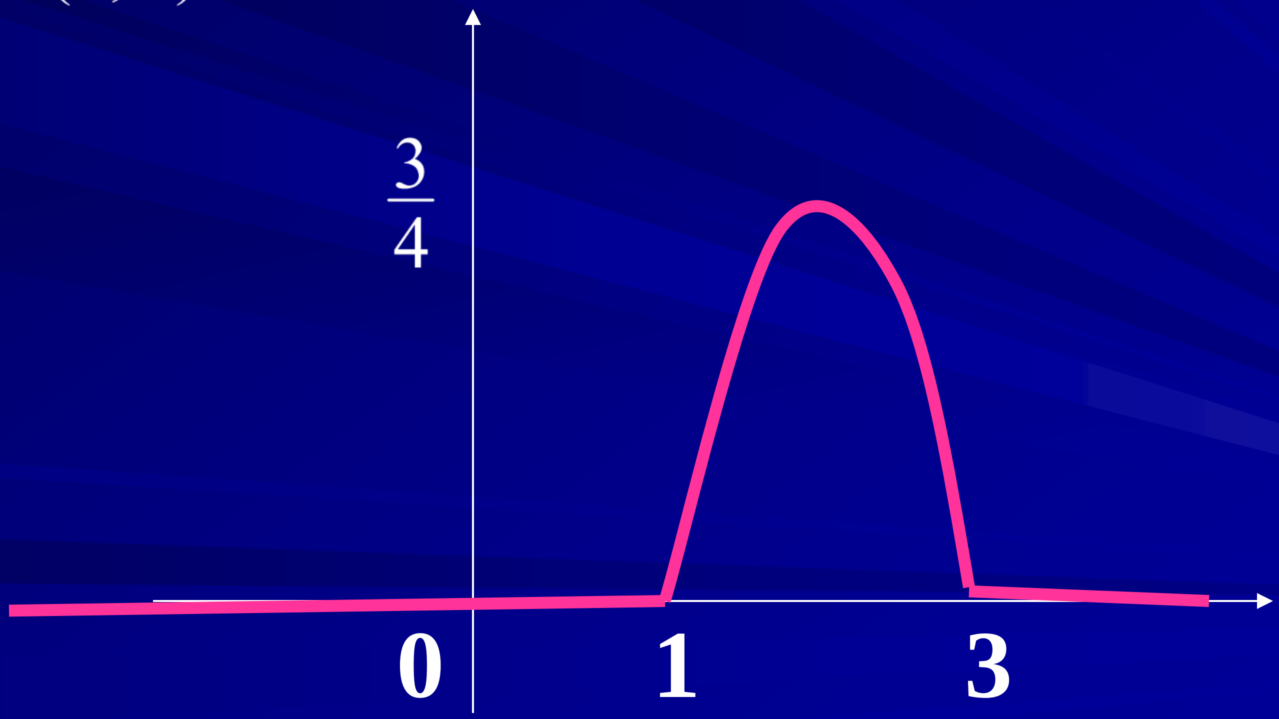
$$2m - 4 = 3$$

$$m = \frac{7}{2}$$

$$f(x)=\begin{cases}\frac{3}{4}(1-(2-x)^2), & 1\le x\le 3\\ 0, & \text{otherwise}\end{cases}$$

$f(1)=0$ $f(3)=0$

$(1,0)$ $(3,0)$



$$E(X)=\int_1^3 \frac{3}{4}(1-(2-x)^2)x\,dx$$

$$=\frac{3}{4}\int_1^3 (-x^3+4x^2-3x)\,dx$$

$$=\frac{3}{4}\left[-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right]_1^3$$

$$=\frac{3}{4}\left[(-\frac{81}{4}+36-\frac{27}{2})-(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2})\right]$$

$$=\frac{3}{4}\left[-\frac{81}{4}+36-\frac{27}{2}+\frac{1}{4}-\frac{4}{3}+\frac{3}{2})\right]$$

$$=\frac{3}{4}\left[-20+36-12-1\frac{1}{3}\right]$$

$$=\frac{3}{4}\times\frac{8}{3}$$

$$=2$$

$$E(X^2)=\int_1^3 \frac{3}{4}(1-(2-x)^2)x^2\,dx$$

$$=\frac{3}{4}\int_1^3 (-x^4+4x^3-3x^2)\,dx$$

$$=\frac{3}{4}\left[-\frac{x^5}{5}+x^4-x^3\right]_1^3$$

$$=\frac{3}{4}\left[(-\frac{243}{5}+81-27)-(-\frac{1}{5}+1-1)\right]$$

$$=\frac{3}{4}\left[-\frac{242}{5}+54\right]$$

$$=\frac{3}{4}\left[\frac{-242+270}{5}\right]$$

$$=\frac{3}{4}\times\frac{28}{5}$$

$$=\frac{21}{5}$$

$$Var(X) = E\left[X^2\right] - [E(X)]^2$$

$$= \frac{21}{5} - 2^2$$

$$= \frac{1}{5}$$

$$\sigma = \sqrt{\frac{1}{5}}$$

Cumulative Distribution Function

$For\ (-\infty < x < 1)$

$$F(x) = 0$$

$For\ (1 \le x \le 3)$

$$F(x) = F(1) + \int_0^x \tfrac{3}{4}(1-(2-x)^2)\,dx = 0 + \frac{3}{4}\int_1^x \left(-x^2+4x-3\right)dx$$

$$= \frac{3}{4}\left[-\frac{x^3}{3}+2x^2-3x\right]_1^x = \frac{3}{4}\left[(-\frac{x^3}{3}+2x^2-3x)-(-\tfrac{1}{3}+2-3)\right]$$

$$= \frac{3}{4}\left[-\frac{x^3}{3}+2x^2-3x+\frac{4}{3}\right]$$

$For\ (3 < x < \infty)$

$$F(x) = F(3)+0 = \frac{3}{4}\left[-\frac{3^3}{3}+2\times 3^2-3\times 3+\frac{4}{3}\right] = 1$$

$$F(x)=\begin{cases} 0 & ,-\infty < x < 1 \\ \frac{3}{4}\left(-\frac{x^3}{3}+2x^2-3x+\frac{4}{3}\right) & ,1 \le x \le 3 \\ 1 & ,\ 3 < x \infty \end{cases}$$

$F(0)=0$ $F(3)=1$