Normal Distribution

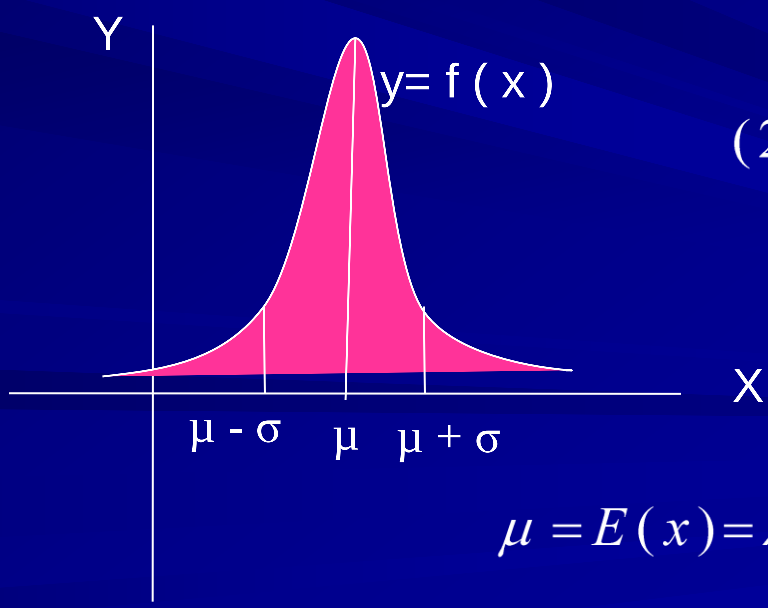
The parameters μ and σ^2 must be fatisfy the condition, $-\infty < \mu < \infty$, $\sigma > 0$ the parameter of the distribution.

If x has normal distribution with parameters μ and σ^2, we use the notation $X \sim n(\mu, \sigma^2)$

$$\text{Graph of } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad , -\infty < x < \infty$$

$$(1)\ f(x) \geq 0$$

$$(2)\ \int_{-\infty}^{\infty} f(x)\,dx = 1$$



$$\mu = E(x) = \lambda \quad , \sigma^2 = Var(x) = \lambda$$

Distribution Function

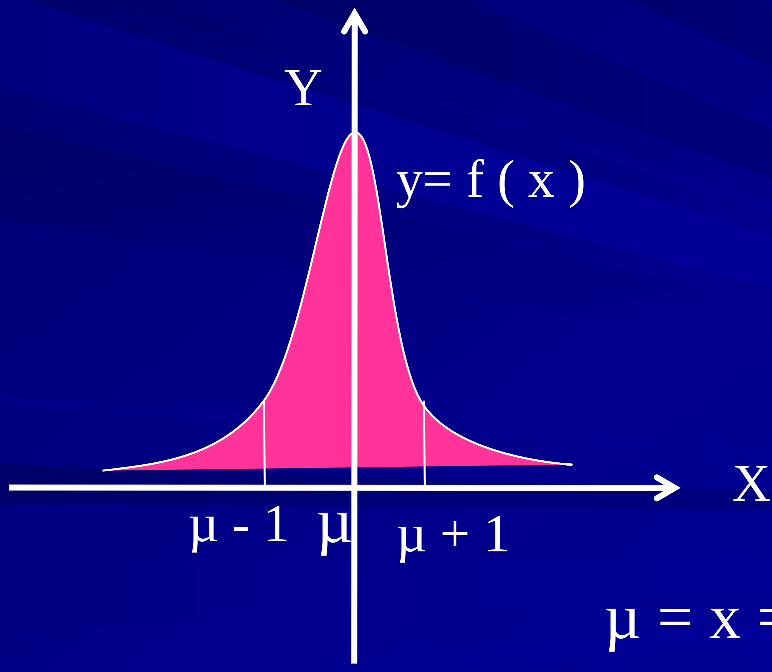
$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(v-\mu)^2}{2\sigma^2}} \, dv$$

$$P(a < x < b) = F(b) - F(a) = \frac{1}{\sigma\sqrt{2\pi}} \int_{a}^{b} e^{\frac{-(v-\mu)^2}{2\sigma^2}} \, dv$$

Standard Normal Distribution

If X has normal distribution $X \sim n(0, 1)$ we say that X has standard normal distribution. That is probability density function of X may be written as

$$f(x) = \frac{1}{\sqrt{2\pi}} \times e^{\frac{-x^2}{2}} \quad , -\infty < x < \infty$$



$$\Phi(x) = \int_{-\infty}^{x} f(x)\,dx$$

$\mu = x = 0$

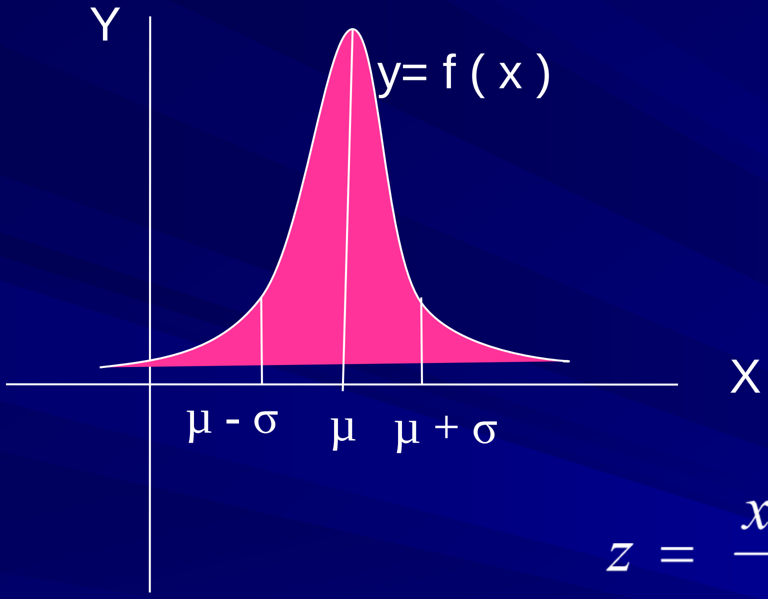
Standard Normal Distribution Function

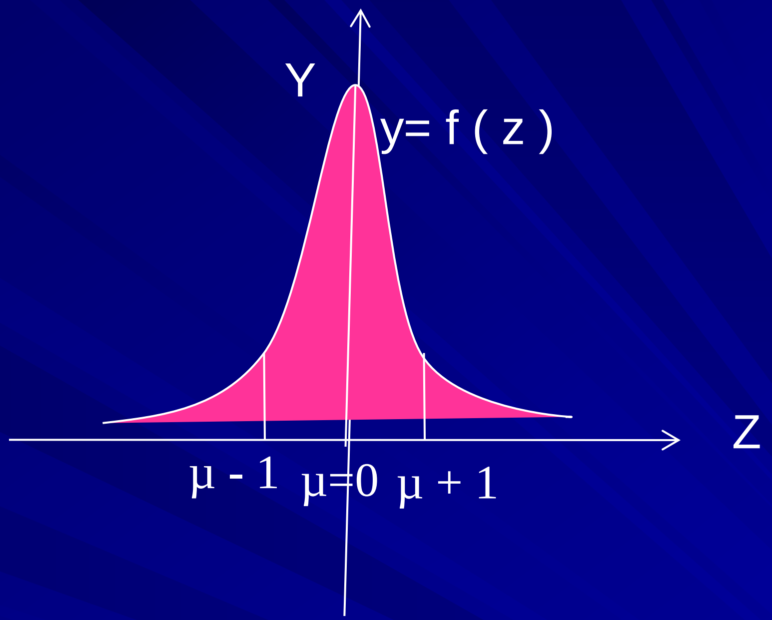
$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{v^2}{2}} \, dv$$

$$P(a < x < b) = F(b) - F(a) = \frac{1}{\sqrt{2\pi}} \int_{a}^{b} e^{-\frac{v^2}{2}} \, dv$$

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{v^2}{2}} \, dv$$

If X has normal distribution $X \sim n(\mu, \sigma^2)$ *and* $Z = \dfrac{X-\mu}{\sigma}$ *then* $Z \sim n(0,1)$

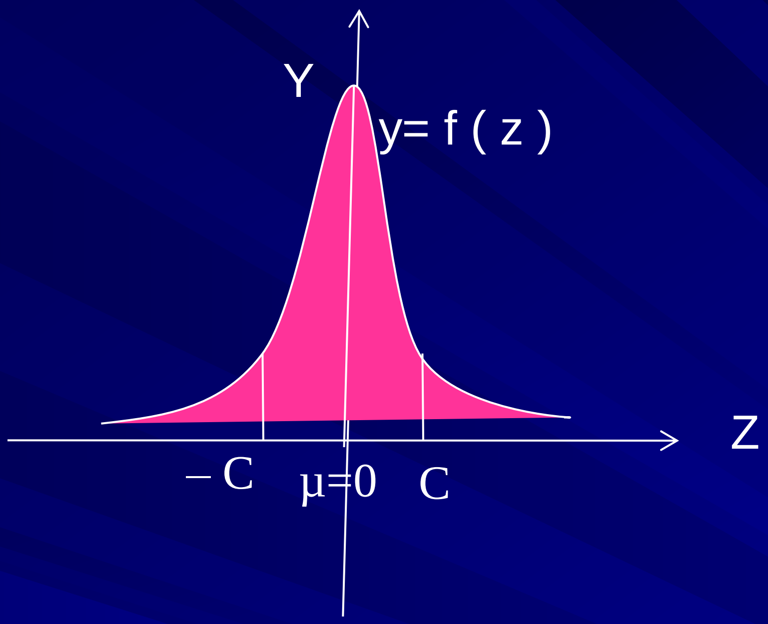




$$z = \frac{x - \mu}{\sigma}$$

$$F\left(\frac{x-\mu}{\sigma}\right) = \Phi(Z)$$

Note –



By the symmetry

$$P(-C < Z < C) = 2\,\Phi(C) - 1$$

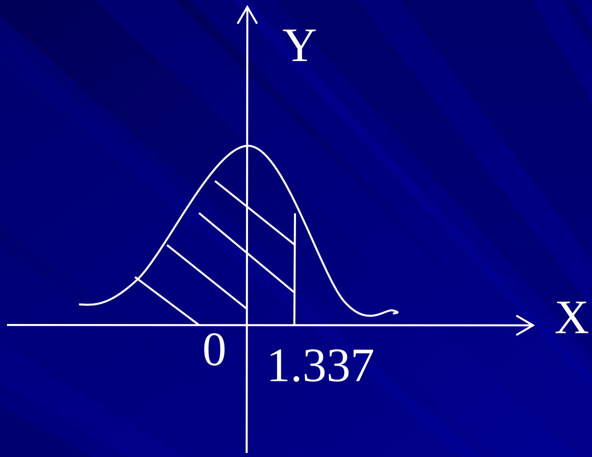
If $X \sim n(0, 1)$, find (i) $P(X < 1.337)$ (ii) $P(X > -1.337)$
(iii) $P(X < -1.337)$ (iv) $P(-2.696 < X < 1.865)$ (v) $P(|X| < 1.433)$
(vi) $P(X > 0.863)$ or $P(X < -1.527)$

If $X \sim n(0, 1)$,

(i) $P(X < 1.337)$

$= \Phi(1.337)$

$= 0.9099$

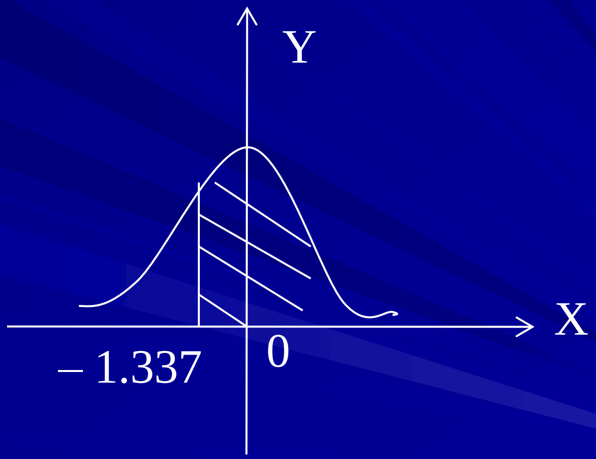


(ii) $P(X > -1.337)$

$= 1 - P(X \leq -1.377)$

$= 1 - \Phi(-1.337) = 1 - 0.0901$

$= 0.9099$

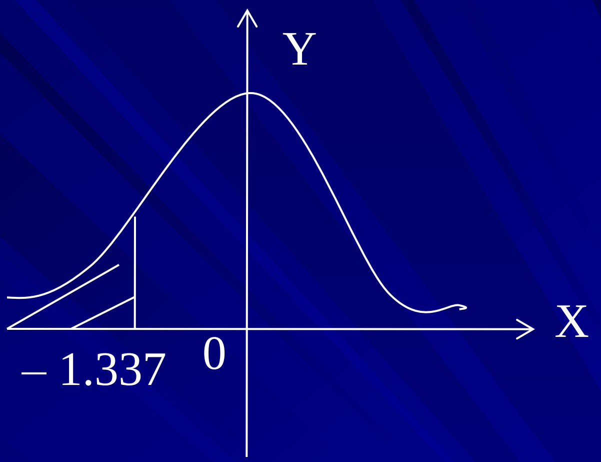


(or) $P(X > -1.337) = P(X < 1.337)$

$= \Phi(1.337) = 0.9099$

(iii) $P(X < -1.337) = \Phi(-1.337)$

$= 0.0901$

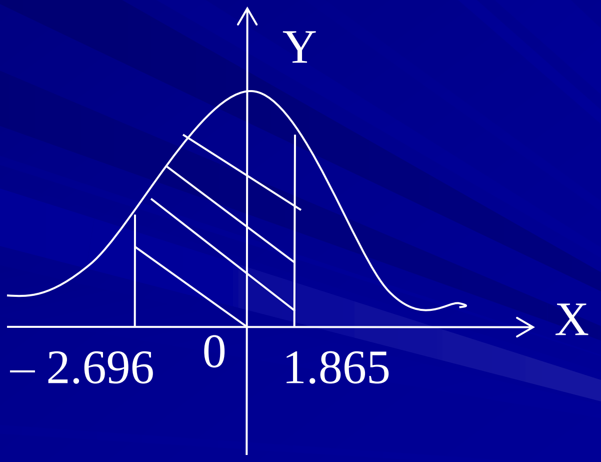


(iv) $P(-2.696 < X < 1.865)$

$= \Phi(1.865) - \Phi(-2.696)$

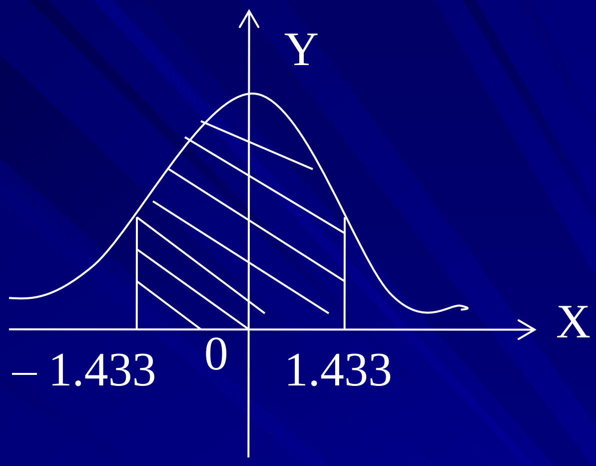
$= 0.9686 - 0.0035$

$= 0.9651$



$(v)\ P(|X| < 1.433)$

$= P(-1.433 < X < 1.433)$



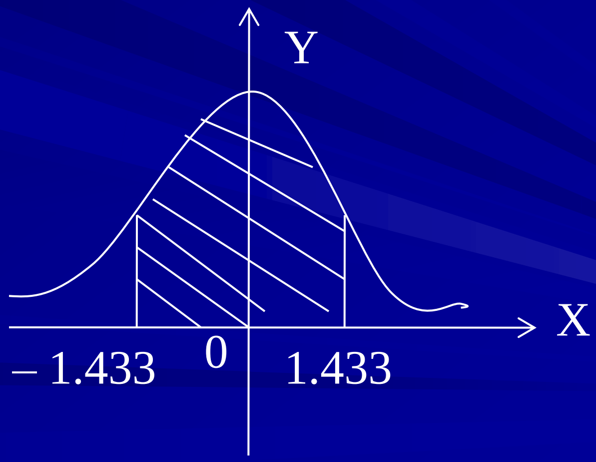
$= \Phi(1.433) - \Phi(-1.433)$

$= 0.9236 - 0.0764$

$= 0.8472$ (or)

$(v)\ P(|X| < 1.433)$

$= P(-1.433 < X < 1.433)$



By the symmetry

$= 2\Phi(1.433) - 1$

$= 2 \times 0.9236 - 1$

$= 0.8472$

(vi) $P(X > 0.863)$ or $P(X < -1.527)$

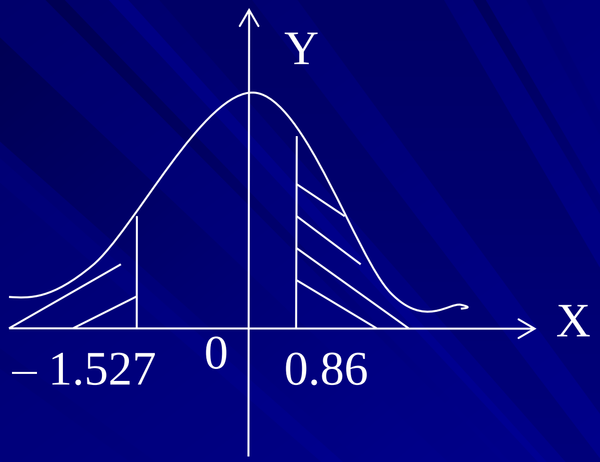
$= P(X > 0.863) + P(X < -1.527)$

$= 1 - P(X \le 0.863) + P(X < -1.527)$

$= 1 - \Phi(0.863) + \Phi(-1.527)$

$= 1 - 0.8051 + 0.0630$

$= 0.2579$



(or)

(vi) $P(X > 0.863)$ or $P(X < -1.527)$

$= 1 - P(-1.527 \le X \le 0.863)$

$= 1 - \{\Phi(0.863) - \Phi(-1.527)\}$

$= 1 - 0.8051 + 0.0630$

$= 0.2579$

If $X \sim n(0, 1)$, find the value of C if (i) $P(X > C) = 0.3802$ (ii) $P(X > C) = 0.7818$ (iii) $P(X < C) = 0.0793$ (iv) $P(X < C) = 0.9693$ (v) $P(|X| < C) = 0.9$

(i) $P(X > C) = 0.3802$

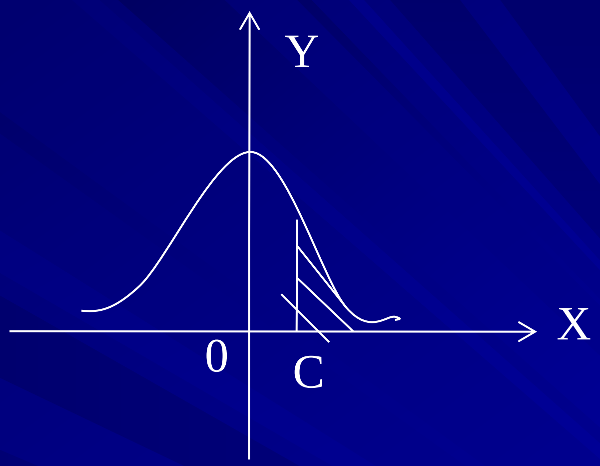
$1 - P(X \leq C) = 0.3802$

$P(X \leq C) = 1 - 0.3802$

$\Phi(C) = 0.6198$

$\Phi(C) = \Phi(0.30)$

$C = 0.3$



(ii) $P(X > C) = 0.7818$

Since, probability is greater than 0.5, C must be negative

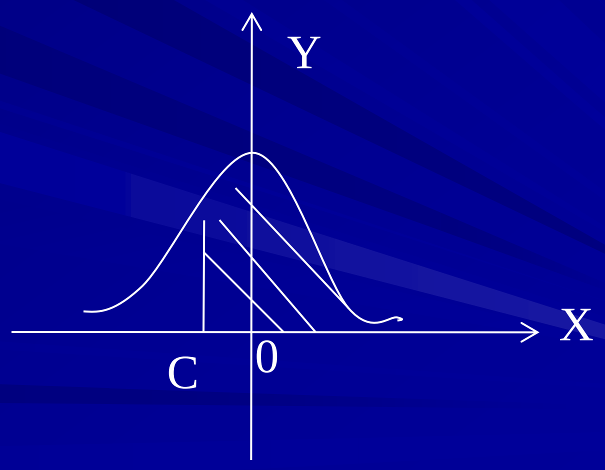
$1 - P(X \leq C) = 0.7818$

$P(X \leq C) = 1 - 0.7818$

$\Phi(C) = 0.2182$

$\Phi(C) = \Phi(-0.78)$

$C = -0.78$

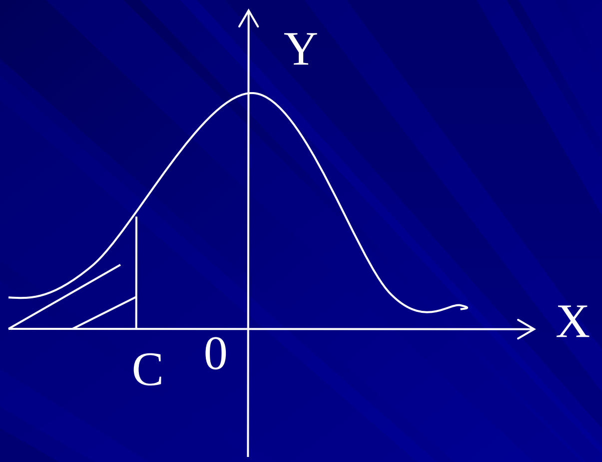


(iii) $P(X < C) = 0.0793$

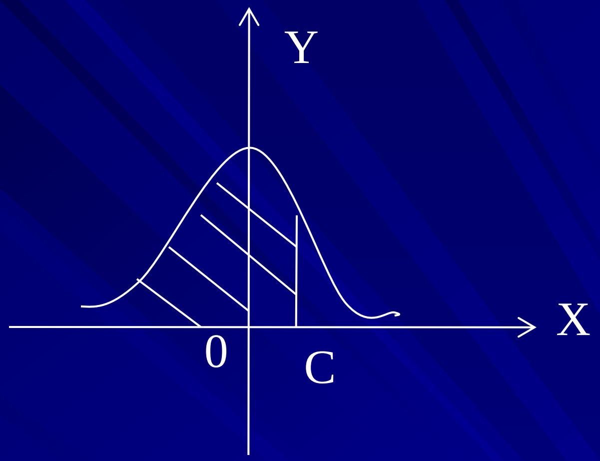
Since, probability is less than 0.5, C must be negative

$\Phi(C) = \Phi(-1.41)$

$C = -1.41$



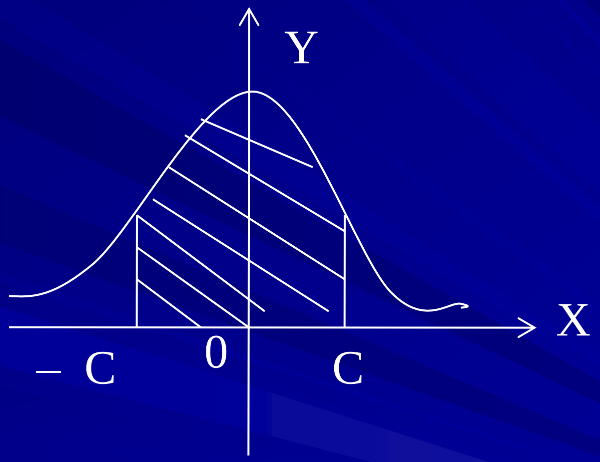
(iv) $P (X < C) = 0.9692$



$\Phi (C) = \Phi (1.87)$

$C = 1.87$

(iii) $P (|X| < C) = 0.9$



i.e, $P (- C < X < C) = 0.9$

By the symmetry

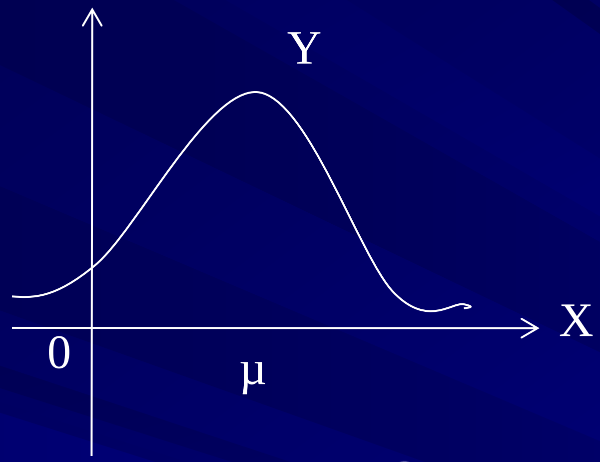
$2 \Phi (C) - 1 = 0.9$

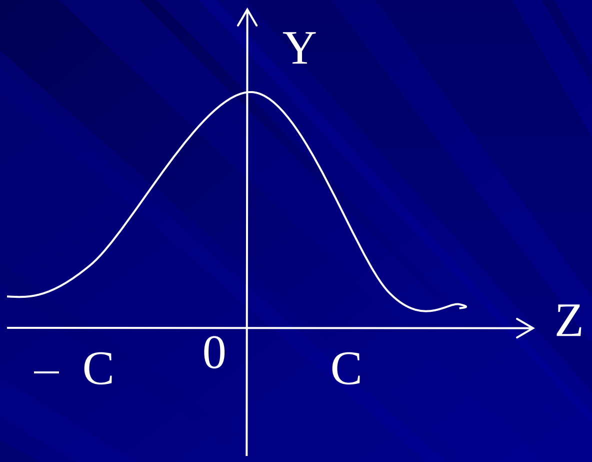
$\Phi (C) = 0.95$

$\Phi (C) = \Phi (1.64)$

$C = 1.64$

Let X be the normal with mean 80 and variance 9. Find P (X > 83) , P (X < 81) , P (X < 80) and P (78 < X < 82)





$$X \sim n(80, 3^2)$$

$$Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 80}{3}$$

$$(i)\, x = 83 \Rightarrow z = \frac{83 - 80}{3} = 1$$

P (X > 83) = P (Z > 1) = 1 – P (Z ≤ 1)

= 1 – Φ (1) = 1 – 0.8413 = 0.1587

$$(ii)\ x = 81 \Rightarrow z = \frac{81-80}{3} = 0.33$$

$$P(X < 81) = P(Z < 0.33)$$

$$= \Phi(0.33)$$

$$= 0.6293$$

$$(iii)\ x = 80 \Rightarrow z = \frac{80-80}{3} = 0$$

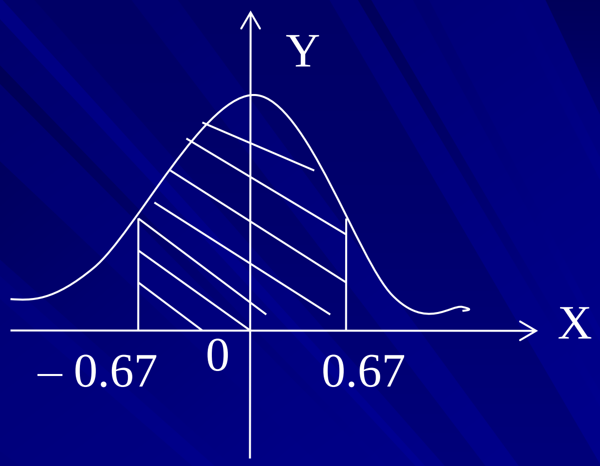
$$P(X < 80) = P(Z < 0)$$

$$= \Phi(0)$$

$$= 0.5$$

$$(iv)\ x = 82 \Rightarrow z = \frac{82-80}{3} = 0.67$$

$$x = 78 \Rightarrow z = \frac{78-80}{3} = -0.67$$



$$P(78 < X < 82) = P(-0.67 < Z < 0.67)$$

$$= 2\,\Phi(0.67) - 1$$ By the symmetry

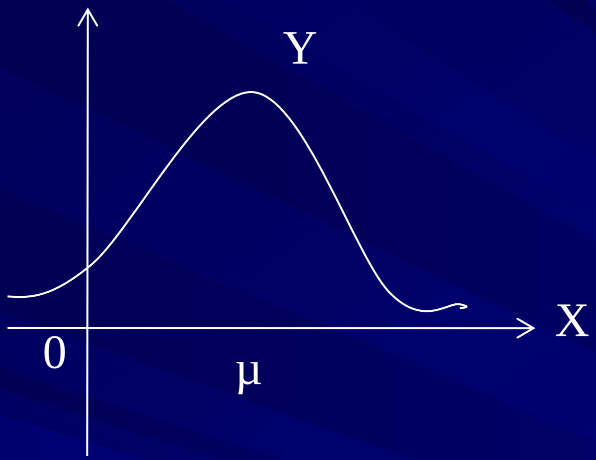
$$= 2 \times 0.7486 - 1$$

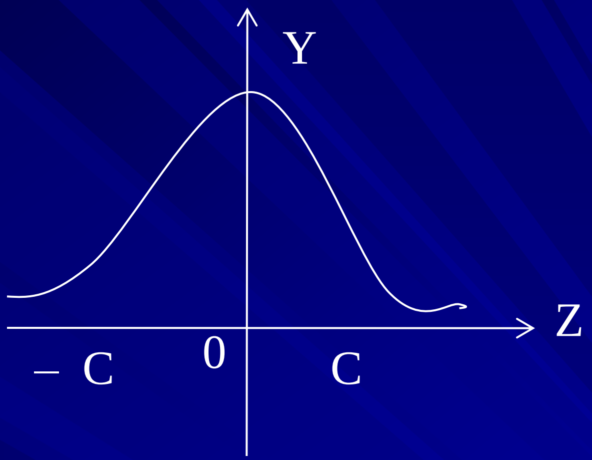
$$= 1.4972 - 1$$

$$= 0.4972$$

(or) $= \Phi(0.67) - \Phi(-0.67) = 0.7486 - 0.2514 = 0.4972$

Let X be the normal with mean 14 and variance 0.01. Determine C such that P (X < C) = 50 % , P (X > C) = 10% and P (– C < X < C) = 99.9%





$$X \sim n(14,(0.1)^2)$$

$$Z \sim n(0,1)$$

$$z = \frac{x-\mu}{\sigma} = \frac{x-14}{0.1}$$

(i)P (X < C) = 0.5

$$(i)\ P(X < C) = 0.5$$

$$P\left(Z < \frac{C-14}{0.1}\right) = 0.5$$

$$\Phi\left(\frac{C-14}{0.1}\right) = \Phi(0)$$

$$\frac{C-14}{0.1} = 0$$

$$C = 14$$

$$(ii)\ z = \frac{x-\mu}{\sigma} = \frac{C-14}{0.1}$$

$$P(X > C) = 0.1$$

$$P\left(Z > \frac{C-14}{0.1}\right) = 0.1$$

$$1 - P\left(Z \le \frac{C-14}{0.1}\right) = 0.1$$

$$P\left(Z \le \frac{C-14}{0.1}\right) = 0.9$$

$$\Phi\left(\frac{C-14}{0.1}\right) = \Phi(1.28)$$

$$\frac{C-14}{0.1} = 1.28$$

$$C = 14.128$$

Let X be normal with mean 3.6 and variance 0.01. Find C such that

(i) $P(X \le C) = 50\%$ (ii) $P(X > C) = 10\%$

(iii) $P(-C < X - 3.6 \le C) = 99.9\%$

$$X \sim n(3.6, (0.1)^2) \qquad Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 3.6}{0.1}$$

(i) $P(X \le C) = 50\%$

$$P\left(Z \le \frac{C - 3.6}{0.1}\right) = 0.5$$

$$\Phi\left(\frac{C - 3.6}{0.1}\right) = \Phi(0)$$

$$\frac{C - 3.6}{0.1} = 0$$

$$C = 3.6$$

(ii) $P(X > C) = 10\%$

$$1 - P(X \le C) = 0.1$$

$$P(X \le C) = 0.9$$

$$P\left(Z \le \frac{C - 3.6}{0.1}\right) = 0.9$$

$$\Phi\left(\frac{C - 3.6}{0.1}\right) = \Phi(1.28)$$

$$\frac{C - 3.6}{0.1} = 1.28$$

$$C = 3.728$$

$$(iii)\ P(-C < X - 3.6 \le C) = 99.9\%$$

$$P(-C + 3.6 < X \le C + 3.6) = 0.999$$

$$P\left(\frac{-C + 3.6 - 3.6}{0.1} < Z \le \frac{C + 3.6 - 3.6}{0.1}\right) = 0.999$$

$$P\left(-\frac{C}{0.1} < Z \le \frac{C}{0.1}\right) = 0.999$$

$$2\Phi\left(\frac{C}{0.1}\right) - 1 = 0.999$$

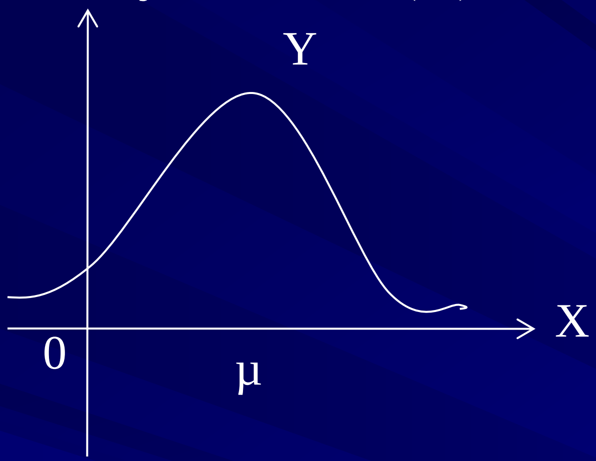
$$\Phi\left(\frac{C}{0.1}\right) = 0.9995$$

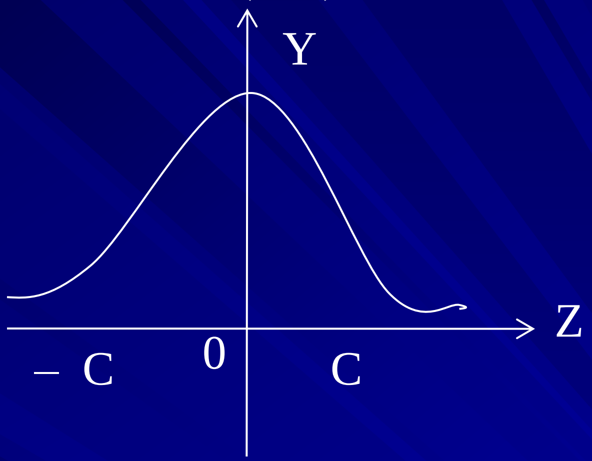
$$\Phi\left(\frac{C}{0.1}\right) = \Phi(3.27)$$

$$\frac{C}{0.1} = 3.27$$

$$C = 0.327$$

For the random variable x of normal distribution X ~ n (10, 25). Find the probability that x is (i) less than 6 (ii) more than 12 (iii) between 3 and 17





$X \sim n(10, 5^2)$

$Z \sim n(0,1)$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 10}{5}$$

$$(i)\, x = 6 \Rightarrow z = \frac{6-10}{5} = -0.8$$

P (X < 6) = P (Z < – 0.8)

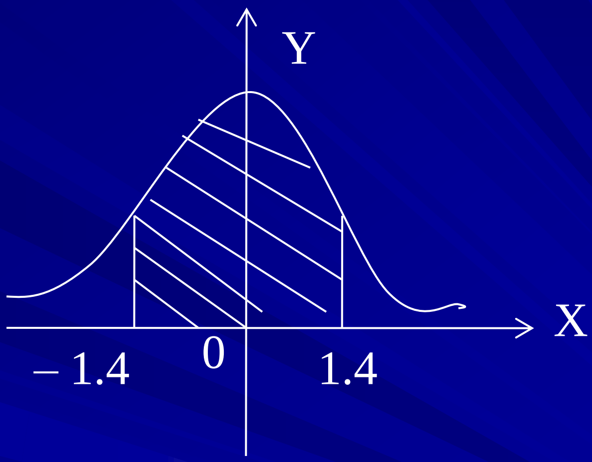
= 0.2119

$$(ii)\ x = 12 \Rightarrow z = \frac{12-10}{5} = 0.4$$

$P(X > 12) = P(Z > 0.4) = 1 - P(Z \leq 0.4) = 1 - \Phi(0.4)$

$= 1 - 0.6554 = 0.3446$

$$(iii)\ x = 3 \Rightarrow z = \frac{3-10}{5} = -1.4$$



$$z = \frac{x-\mu}{\sigma} = \frac{17-10}{5} = 1.4$$

$P(3 < X < 17) = P(-1.4 < Z < 1.4)$

$= 2\,\Phi(1.4) - 1$ (By the smmstry)

$= 2 \times 0.9192 - 1$

$= 0.8384$

7. Suppose that hight of 800 students are normally distribution with mean 66 inches and student deviation 5 inches. Find the number of students with hight (i) between 65 and 70 inches (ii) greater than or equal 6 ft.

X be the height of students

$X \sim n(66, 5^2)$

$Z \sim n(0,1)$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 66}{5}$$

$$(i)\, x = 65 \Rightarrow z = \frac{65 - 66}{5} = -0.2$$

$$x = 70 \Rightarrow z = \frac{70 - 66}{5} = 0.8$$

$$(i)\ P (65 < X < 70) = P (-0.2 < Z < 0.8)$$

$$= \Phi (0.8) - \Phi (-0.2)$$

$$= 0.7881 - 0.4207$$

$$= 0.3674$$

The number of students with height between 65 and 70 inches

$$= 0.3674 \times 800 = 293.92 = 294$$

$$(ii)\ x = 72 \Rightarrow z = \frac{72-66}{5} = 1.2$$

$$P(X \geq 6\text{ ft}) = P(X \geq 72\text{ inches}) = P(Z \geq 1.2)$$

$$= 1 - P(Z < 1.2)$$

$$= 1 - \Phi(1.2)$$

$$= 1 - 0.8849$$

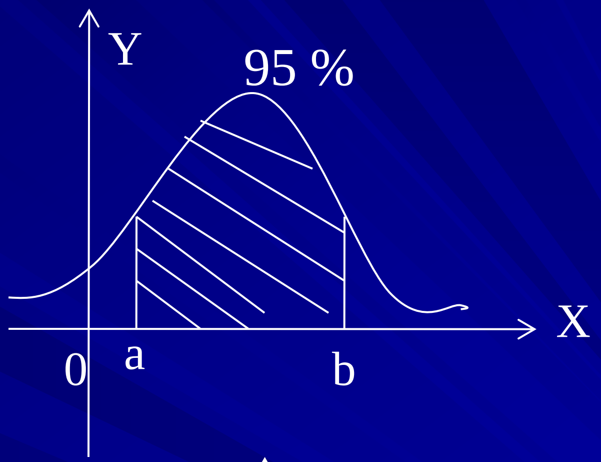
$$= 0.1151$$

The number of students with height greater than or equal 6 ft

$$= 0.1151 \times 800 = 92.08$$

(iii) Find the range symmetrical about the mean, within which 95 % of the students lie.

Consider, the range symmetrical about the mean, within which 95 % of the students lie height between a inches and b inches.



i.e, $P(a < X < b) = P(-C < Z < C) = 0.95$

where $\frac{a - 66}{5} = -C$, $\frac{b - 66}{5} = C$

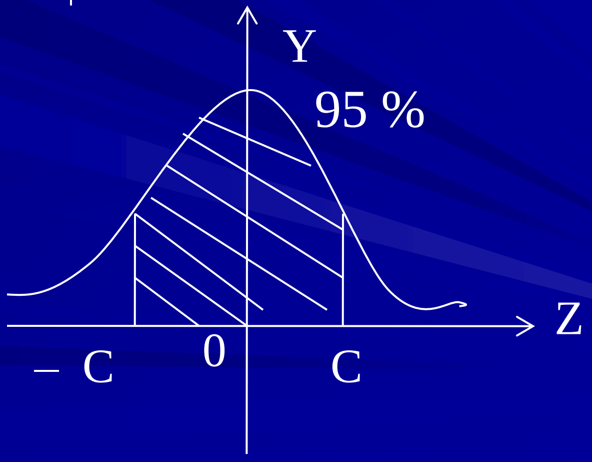
$P(-C < Z < C) = 0.95$

$2\Phi(C) - 1 = 0.95$

$\Phi(C) = 0.975$

$\Phi(C) = \Phi(1.96)$

$C = 1.96$ For Z



Therefore, for X

$$\frac{a-66}{5}=-1.96 \quad , \quad \frac{b-66}{5}=1.96$$

$$a=56.2 \quad , \quad b=75.8$$

The range symmetrical about the mean, within which 95 % of the students lie height between 56.2 inches and 75.8 inches .

8. Suppose that the diameters of bolt manufactored by a company are normally distribution with mean 0.25 inches and standare deviation 0.02 inches. A bolt is consided defective if its diameter is lass than equal to 0.2 or grether than 0.28 inches . Find the percentage of defective bolts manufactored by the company.

X be the diameter of bolts

$$X \sim n(0.25,(0.02)^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x-\mu}{\sigma} = \frac{x-0.25}{0.02}$$

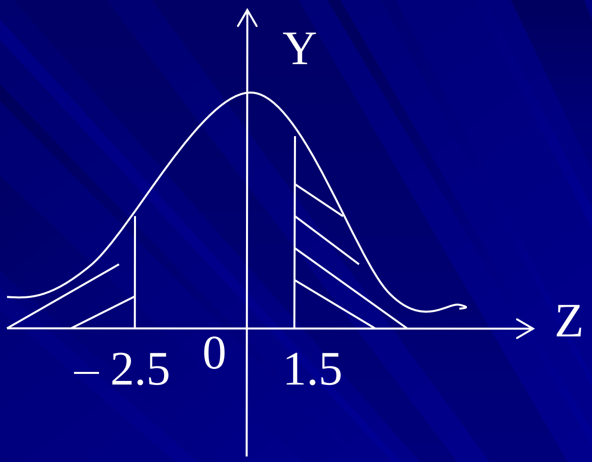
$$x = 0.2 \Rightarrow z = \frac{0.2 - 0.25}{0.02} = -2.5$$

$$x = 0.28 \Rightarrow z = \frac{0.28 - 0.25}{0.02} = 1.5$$

$P(X \leq 0.2)$ or $P(X > 0.28) = P(Z \leq -2.5)$ or $P(Z > 1.5)$

$= P(Z \leq -2.5) + P(Z > 1.5)$

$= P(Z \leq -2.5) + 1 - P(Z \leq 1.5)$



$= \Phi(-2.5) + 1 - \Phi(1.5)$

$= 0.0062 + 1 - 0.9332$

$= 0.073$

(OR) $P(X \leq 0.2)$ or $P(X < 0.28)$

$= 1 - \{P(0.2 < X \leq 0.28)\}$

$= 1 - \{P(-2.5 < Z \leq 1.5)\}$

$= 1 - \{\Phi(1.5) - \Phi(-2.5)\}$

$= 1 - 0.9332 + 0.0062$

$= 0.073$

Parcentage of defective bolts are 7.3%

(iii) Find the range symmetrical about the mean, within which 90 % of the bolts lie.

Let $P(a < X < b) = P(-C < Z < C) = 0.9$

where $\dfrac{a - 0.25}{0.02} = -C$, $\dfrac{b - 0.25}{0.02} = C$

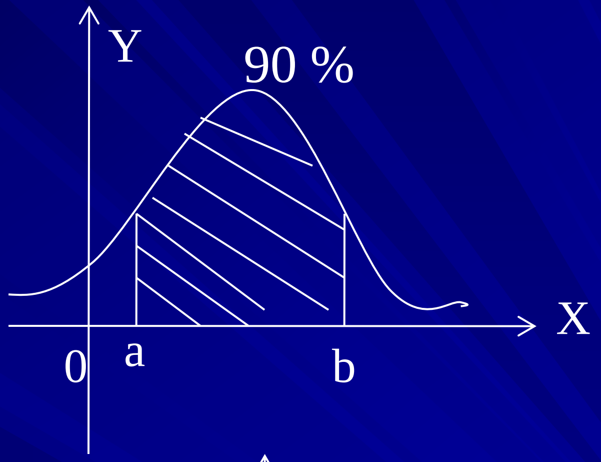
$P(-C < Z < C) = 0.9$

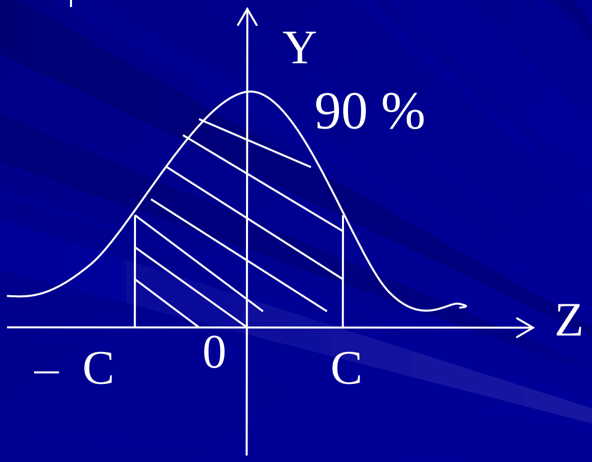
$2\,\Phi(C) - 1 = 0.9$

$\Phi(C) = 0.95$

$\Phi(C) = \Phi(1.64)$

$C = 1.64$ For Z





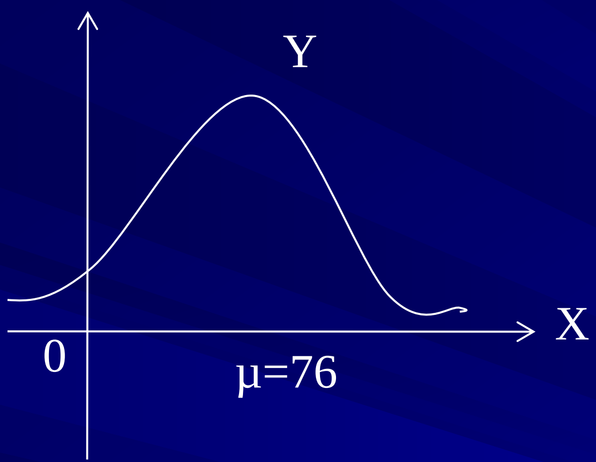
Therefore, for X

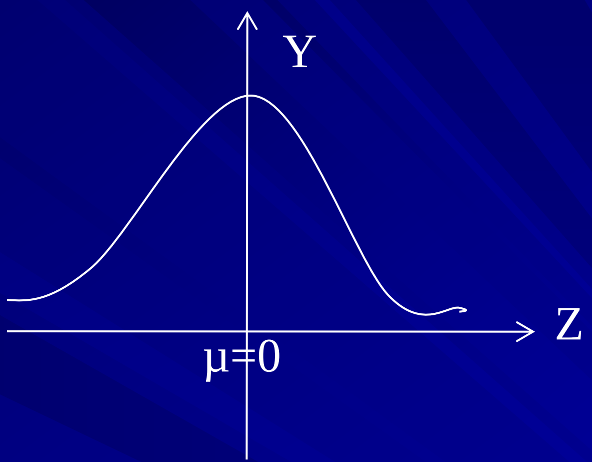
$$\frac{a - 0.25}{0.02} = -1.64 \quad , \quad \frac{b - 0.25}{0.02} = 1.64$$

$$a = 0.2172 \quad , \quad b = 0.2828$$

The range symmetrical about the mean, within which 90 % of the bolts lie diameter between 0.2172 inches and 0.2828 inches .

9. Suppose that the scores on an examination are normally distribution with mean 76 and standard deviation 15. The top 15 % of the students receive A's and button 10 % receive F's. Find the minimum score to receive an A (ii) minimum score to pass (not to receive F)





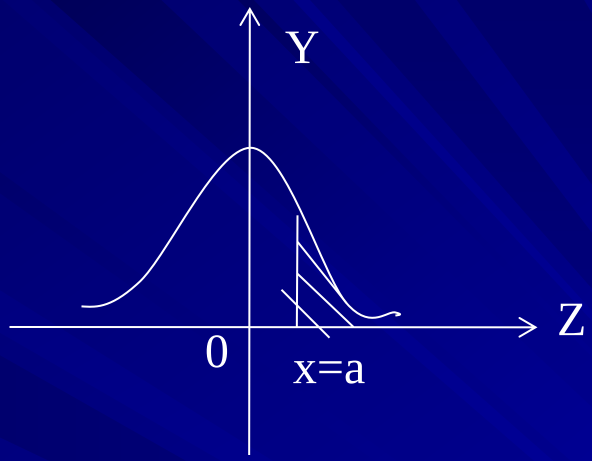
X be the scores of students on an examination

$$X \sim n(76,(15)^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x-\mu}{\sigma} = \frac{x-76}{15}$$

(i) “a” be the minimum scores to receive A’s.

$$x = a \Rightarrow z = \frac{a - 76}{15}$$



$$P(X \geq a) = 15\% = 0.15$$

$$P\left(Z \geq \frac{a - 76}{15}\right) = 0.15$$

$$1 - P\left(Z \leq \frac{a - 76}{15}\right) = 0.15$$

$$\Phi\left(\frac{a - 76}{15}\right) = 0.85$$

$$\Phi\left(\frac{a - 76}{15}\right) = \Phi(1.04)$$

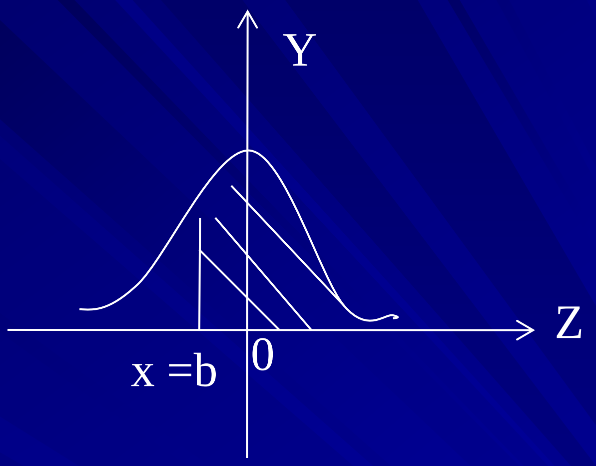
$$\frac{a - 76}{15} = 1.04$$

$$a = (1.04 \times 15) + 76$$

$$a = 91.6$$

(i) "b" be the minimum scores to pass.

$$x = b \Rightarrow z = \frac{b - 76}{15}$$



$$P(X < b) = 10\%$$

$$P\left(Z < \frac{b - 76}{15}\right) = 0.1$$

$$\Phi\left(\frac{b - 76}{15}\right) = \Phi(-1.28)$$

$$\frac{b-76}{15} = -1.28$$

$$b = (-1.28 \times 15) + 76$$

$$b = 56.8$$

(iii) Find the range symmetrical about the mean, within which 75 % of the students lie.

let $P(a < X < b) = P(-C < Z < C) = 0.75$

where $\frac{a - 76}{15} = -C$, $\frac{b - 76}{15} = C$

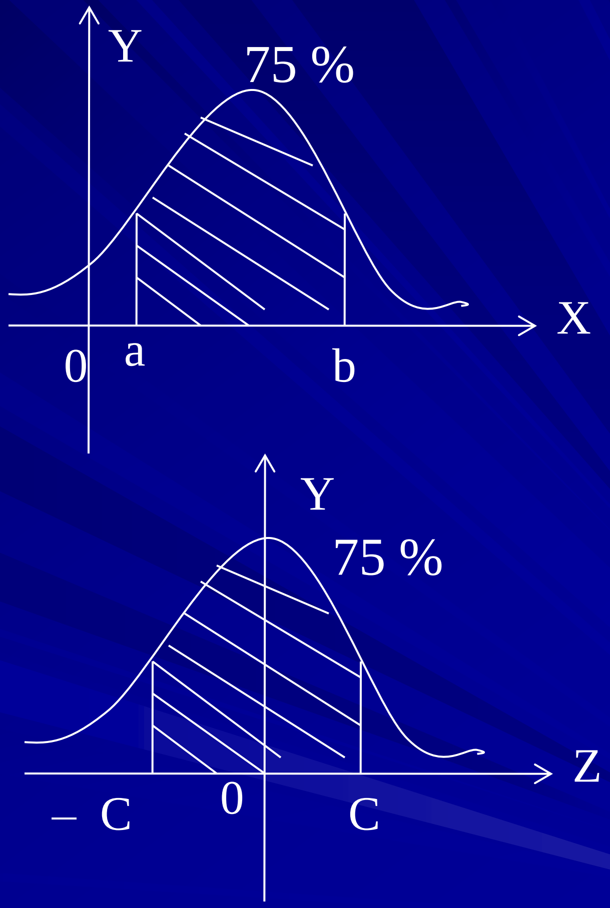
$P(-C < Z < C) = 0.75$

$2\Phi(C) - 1 = 0.75$

$\Phi(C) = 0.875$

$\Phi(C) = \Phi(1.15)$

$C = 1.15$ For Z



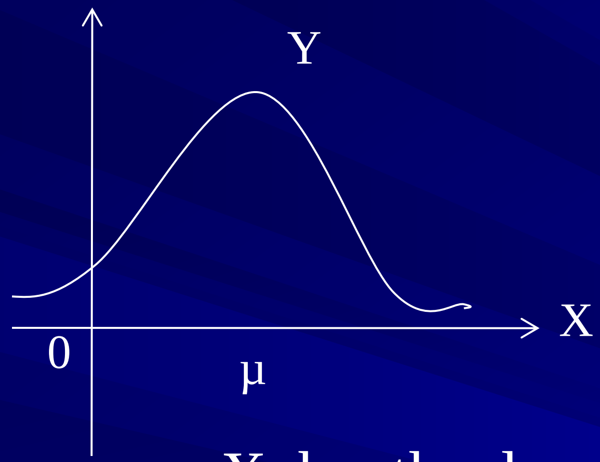
Therefore, for X

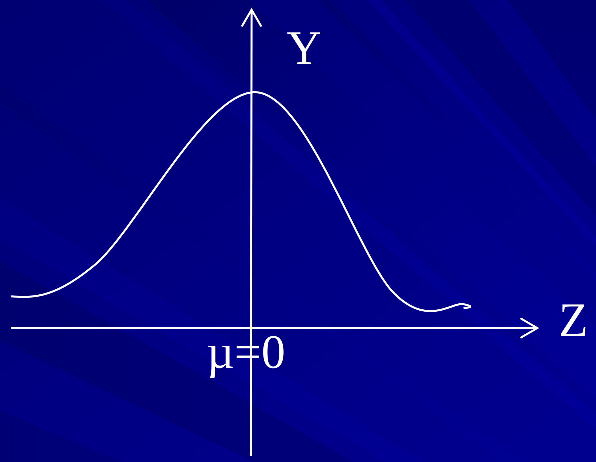
$$\frac{a - 76}{15} = -1.15 \quad , \quad \frac{b - 76}{15} = 1.15$$

$$a = 58.75 \quad , \quad b = 93.25$$

The range symmetrical about the mean, within which 75 % of the students lie marks between 58.75 marks and 93.25 marks .

The length of certain items follow a normal distribution with the mean μ cm and standard deviation 6 cm. It is known that 4.78 % of the items have a length greater than 82 cm. Find the value of the mean μ.





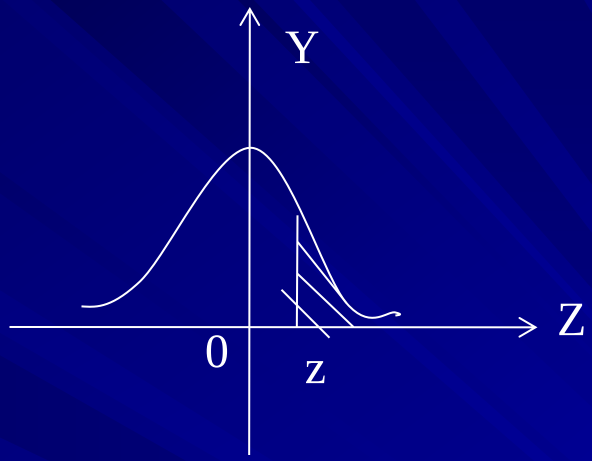
X be the length of items

$$X \sim n(\mu, 6^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x-\mu}{\sigma} = \frac{x-\mu}{6}$$

$$x = 82 \Rightarrow z = \frac{82-\mu}{6}$$

$$P(X > 82) = 4.78\% = 0.0478$$



$$P\left(Z > \frac{82-\mu}{6}\right) = 0.0478$$

$$1 - P\left(Z \le \frac{82-\mu}{6}\right) = 0.0478$$

$$P\left(Z \le \frac{82-\mu}{6}\right) = 0.9522$$

$$\Phi\left(\frac{82-\mu}{6}\right) = 0.9522$$

$$\Phi\left(\frac{82-\mu}{6}\right) = \Phi\,(1.67)$$

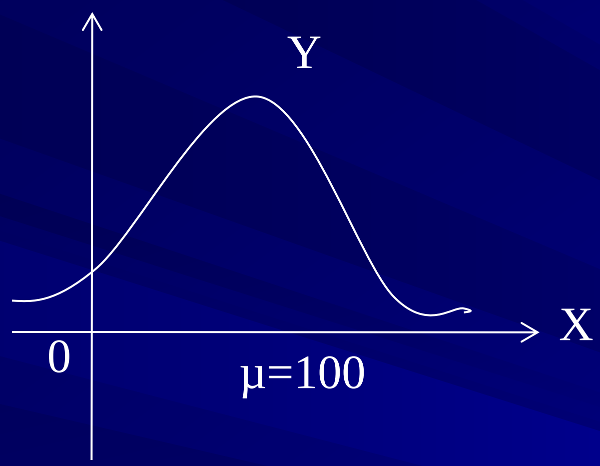
$$\frac{82-\mu}{6} = 1.67$$

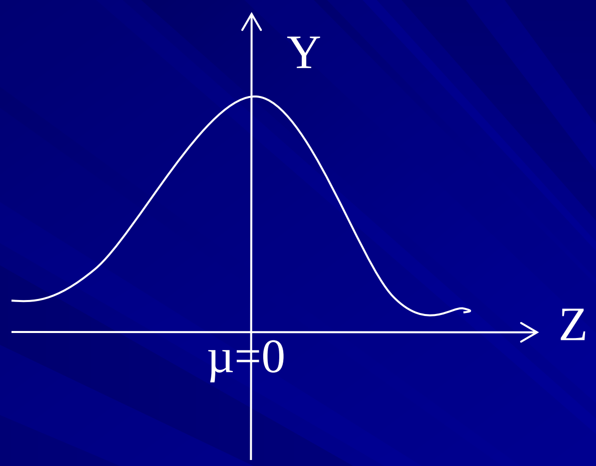
$$\mu = 82 - (1.67 \times 6)$$

$$\mu = 82 - 10.02$$

$$\mu = 71.98$$

X ~ n (100 , σ^2) and P (X < 106) = 0.8849. Find the standard deviation σ.

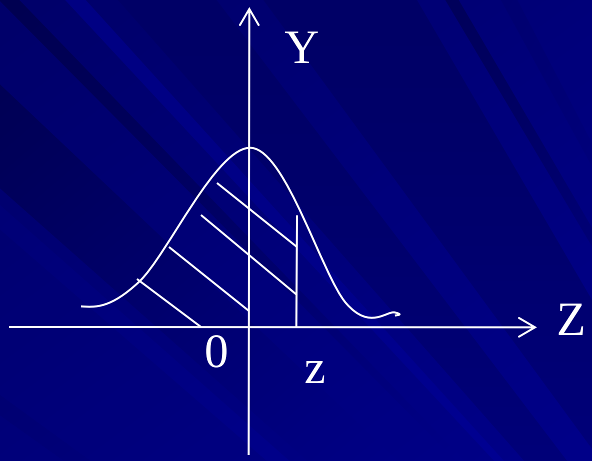




$$X \sim n(100, \sigma^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 100}{\sigma}$$

$$x = 106 \Rightarrow z = \frac{106 - 100}{\sigma} = \frac{6}{\sigma}$$



$$P(X < 106) = 0.8849$$

$$P\left(Z < \frac{6}{\sigma}\right) = 0.8849$$

$$\Phi\left(\frac{6}{\sigma}\right) = \Phi(1.2)$$

$$\frac{6}{\sigma} = 1.2$$

$$\sigma = 5$$

The masses of articles produced in particular workshop are normally distributed with mean μ and standard deviation σ. 8.08 % of articles have a mass greater than 85g and 5.48 % have a mass less than 25g. Find the value of μ and σ, and find the range symmetrical about the mean, within which 75 % of the mass lie.

X be the masses of articles

$$X \sim n(\mu, \sigma^2), \; Z \sim n(0,1)$$

$$z = \frac{x-\mu}{\sigma}$$

$$x = 85 \Rightarrow z = \frac{85-\mu}{\sigma} \qquad x = 25 \Rightarrow z = \frac{25-\mu}{\sigma}$$

$$P(X > 85) = 0.0808 \qquad P(X < 25) = 0.0548$$

$$P(X > 85) = 0.0808$$

$$1 - P\left(Z \leq \frac{85 - \mu}{\sigma}\right) = 0.0808$$

$$P\left(Z \leq \frac{85 - \mu}{\sigma}\right) = 0.9192$$

$$\Phi\left(\frac{85 - \mu}{\sigma}\right) = \Phi(1.4)$$

$$\frac{85 - \mu}{\sigma} = 1.4$$

$$\mu = 85 - 1.4\sigma \quad \_\_\_(1)$$

$$P(X < 25) = 0.0548$$

$$P\left(Z < \frac{25 - \mu}{\sigma}\right) = 0.0548$$

$$\Phi\left(\frac{25 - \mu}{\sigma}\right) = \Phi(-1.6)$$

$$\frac{25 - \mu}{\sigma} = -1.6$$

$$\mu = 25 + 1.6\sigma \quad \_\_\_(2)$$

By eq (1) and eq (2)

$$85 - 1.4\,\sigma = 25 + 1.6\,\sigma$$

$$3\,\sigma = 60$$

$$\sigma = 20$$

Substitute in eq (1)

$$\mu = 85 - (1.4 \times 20)$$

$$\mu = 57$$

Let $P(a < X < b) = P(-C < Z < C) = 0.75$

where $\dfrac{a - 57}{20} = -C \ , \dfrac{b - 57}{20} = C$

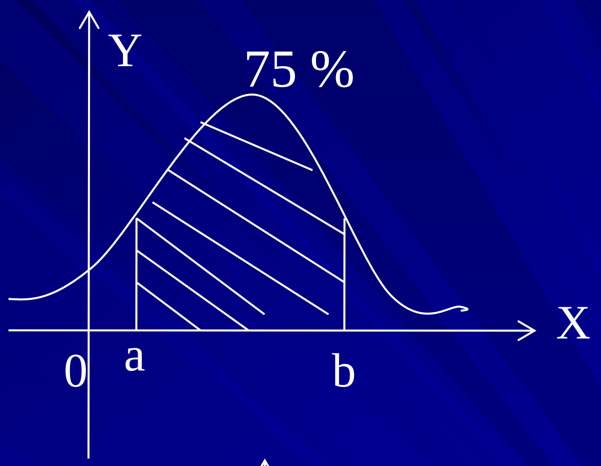
$P(-C < Z < C) = 0.75$

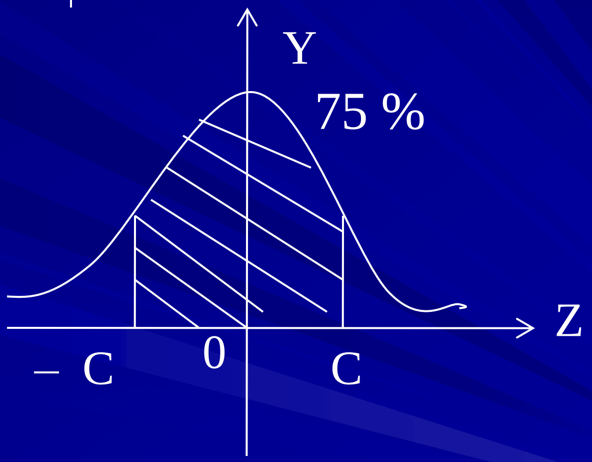
$2\,\Phi(C) - 1 = 0.75$

$\Phi(C) = 0.875$

$\Phi(C) = \Phi(1.15)$

$C = 1.15$ For Z





Therefore, for X

$$\frac{a-57}{20} = -1.15 \quad , \quad \frac{b-57}{20} = 1.15$$

$$a = 34 \quad , \quad b = 80$$

Therefore, central 75 % of distribution lies between the limit 34g and 80g

For another subject (1 29 year-olds meal) in the study by Diskin et al. (A- 10) , acetone level were normally distributed with a mean of 870 and a standard deviation of 200 ppb. Find the probability that on given day the subject's acetone level is (i) between 600 and 1000 ppb (ii) over 500 ppb (iii) between 900 and 1100 ppb.

X be the acetone level

$$X \sim n(870, 200^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 870}{200}$$

$$(\text{i})\ x = 600 \Rightarrow z = \frac{600 - 870}{200} = -1.35$$

$$x = 1000 \Rightarrow z = \frac{1000 - 870}{200} = 0.65$$

$$P(600 < X < 1000) = P(-1.35 < Z < 0.65)$$

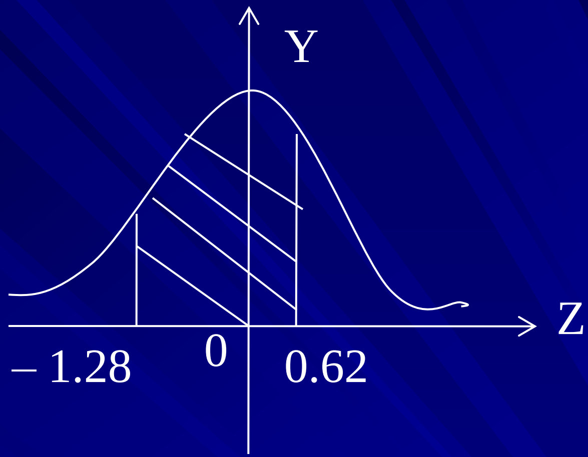
$P(600 < X < 1000)$

$= P(-1.35 < Z < 0.65)$

$= \Phi(0.65) - \Phi(-1.35)$

$= 0.7422 - 0.0885$

$= 0.6537$



(ii) $x = 500 \Rightarrow z = \dfrac{500 - 870}{200} = -1.85$

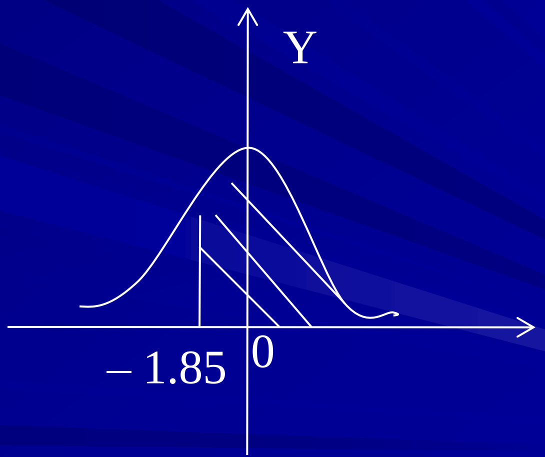
$P(x > 500) = P(Z > -1.85)$

$= 1 - P(Z \leq -1.85)$

$= 1 - \Phi(-1.85)$

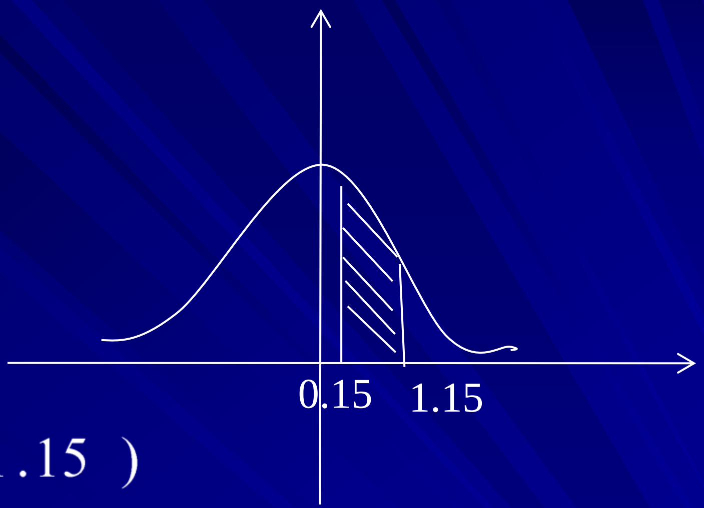
$= 1 - 0.0322$

$= 0.9678$



$$(iii)\ x = 900 \Rightarrow z = \frac{900 - 870}{200} = 0.15$$

$$x = 1100 \Rightarrow z = \frac{1100 - 870}{200} = 1.15$$



$$P(900 < X < 1100) = P(0.15 < Z < 1.15)$$

$$= \Phi(1.15) - \Phi(0.15)$$

$$= 0.8749 - 0.5596$$

$$= 0.3153$$

In the study of fingerprints an important quantitative characteristic is the total ridge count for the 10 fingers of an individual. Suppose that the total ridge counts of individuals in a certain population are approximately normally distributed with a mean of 140 and standard deviation of 50. Find the probability that an individual picked at random from the population will have a ridge count of; (i) 200 or more (ii) less than 100 (iii) between 100 and 200 (iv) between 200 and 250.

X be the ridge

$$X \sim n(140, 50^2),\ Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 140}{50}$$

$$(\mathrm{i})\quad x = 200 \Rightarrow z = \frac{200 - 140}{50} = 1.25$$

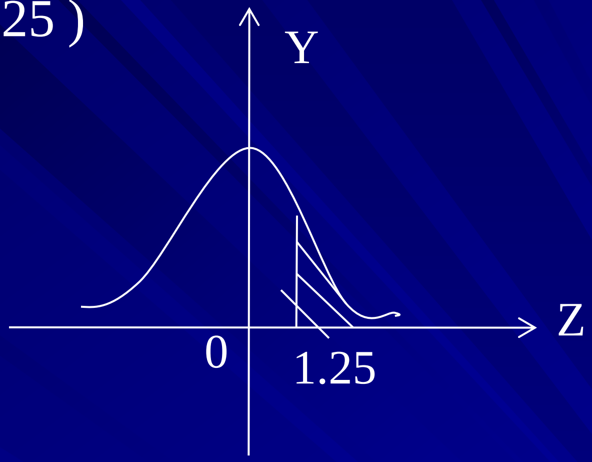
$$P(X \geq 200) = P(Z \geq 1.25)$$

$P(X \geq 200) = P(Z \geq 1.25) = 1 - P(Z < 1.25)$

$= 1 - \Phi(1.25)$

$= 1 - 0.8944$

$= 0.1056$

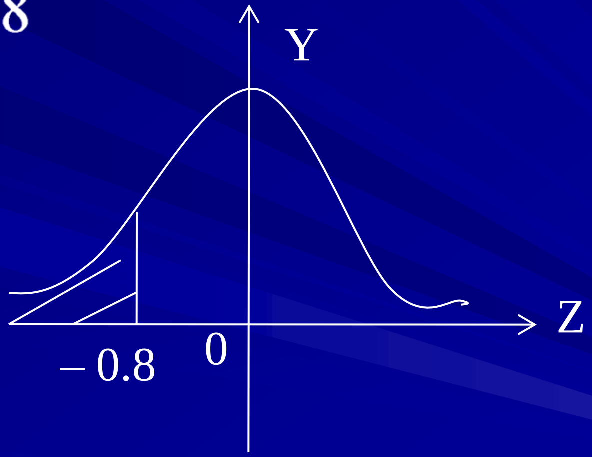


$$\text{(ii)}\ X = 100 \Rightarrow z = \frac{100 - 140}{50} = -0.8$$

$P(X < 100) = P(Z < -0.8)$

$= \Phi(-0.8)$

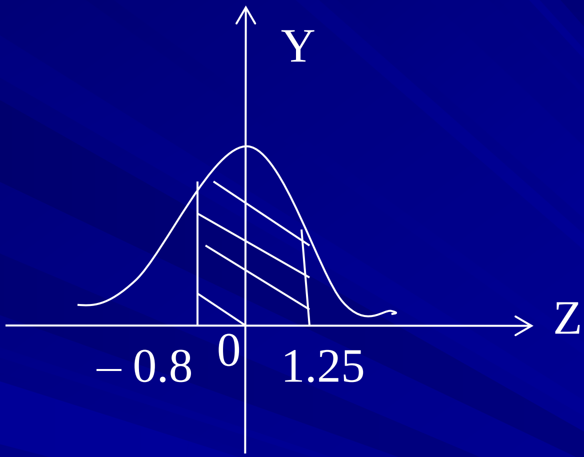
$= 0.2119$



(iii) $P(100 < X < 200)$

$$x = 100 \Rightarrow z = \frac{100 - 140}{50} = -0.8$$

$$x = 200 \Rightarrow z = \frac{200 - 140}{50} = 1.25$$



$P(100 < X < 200)$

$= P(-0.8 < Z < 1.25)$

$= \Phi(1.25) - \Phi(-0.8)$

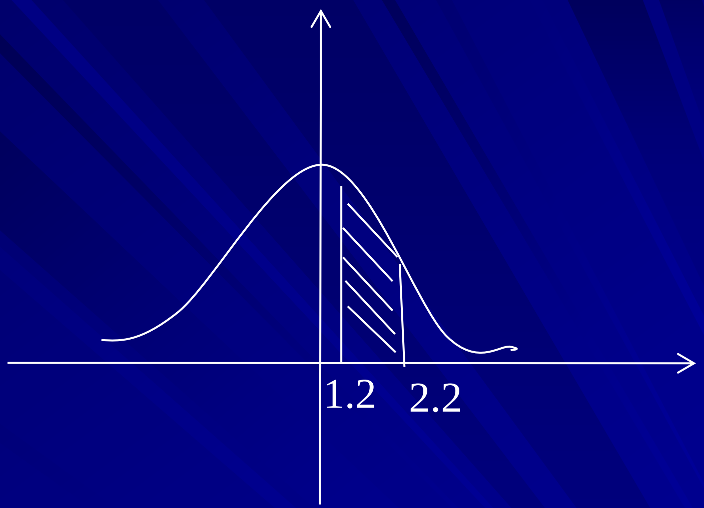
$= 0.8944 - 0.2119$

$= 0.6825$

(iii) P (200 < X < 250)

$$x = 200 \Rightarrow z = \frac{200 - 140}{50} = 1.2$$

$$x = 250 \Rightarrow z = \frac{250 - 140}{50} = 2.2$$



P (200 < X < 250)

= P (1.2 < Z < 2.2)

= Φ (2.2) – Φ (1.2)

= 0.9861 – 0.8849

= 0.1012

On the variable collected in the North Corolina Brith Registry data (A-6) is pounds gained during pregnancy. According to data from the entire for 2001, the number of pound ganied during preganacy was approximately normally distributed with a mean of 30 pounds and standard deviation of 12 pounds. Calculate the probability that a randomly selected morther in North Carolina 2001 gained; (i) Less than 15 pounds during pregnanacy (ii) more than 50 pounds (iii) Between 14 and 40 pounds (iv) Less than 10 pounds (v) Between 10 and 20 pounds.

X be the pounds gained during pregnancy

$$X \sim n(30, 12^2) \,,\; Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 30}{12}$$

(i) P (X < 15)

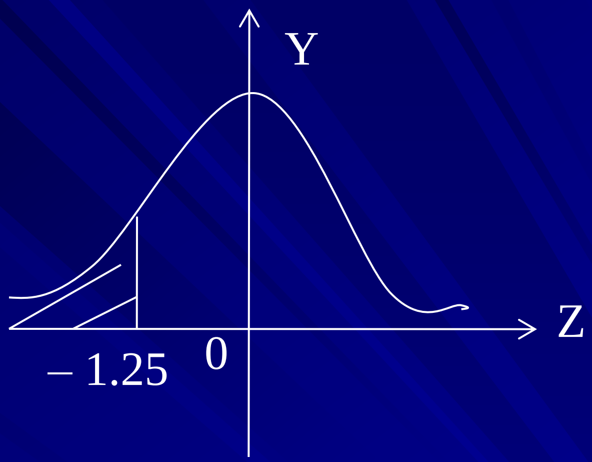
$$x = 15 \Rightarrow z = \frac{15 - 30}{12} = -1.25$$

$$P(X < 15) = P(Z < -1.25)$$

$$P(X < 15) = P(Z < -1.25)$$

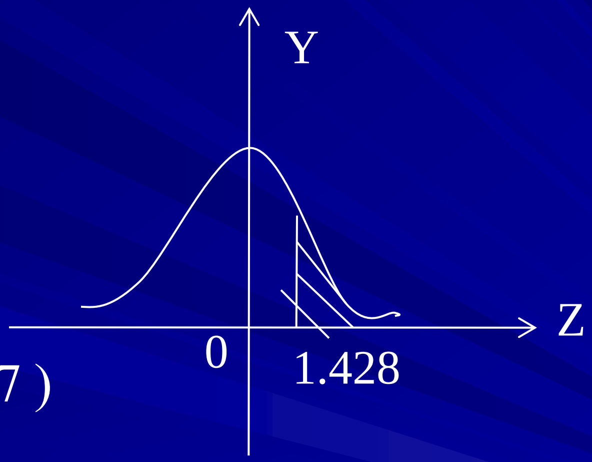
$$= \Phi(-1.25)$$

$$= 0.1056$$



(ii) $P(X > 50)$

$$x = 30 \Rightarrow z = \frac{50 - 30}{12} = 1.67$$



$$P(X > 50) = P(Z > 1.67) = 1 - P(Z \leq 1.67)$$

$$= 1 - \Phi(1.67)$$

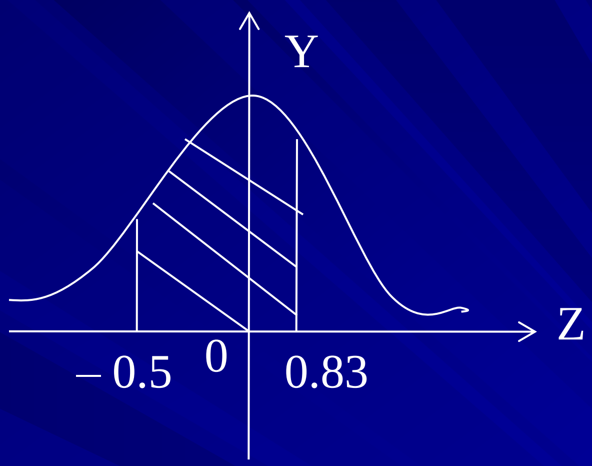
$$= 1 - 0.9525$$

$$= 0.0475$$

(iii) P(14 < X < 40)

$$x = 14 \Rightarrow z = \frac{14 - 30}{12} = -0.5$$

$$x = 40 \Rightarrow z = \frac{40 - 30}{12} = 0.83$$



P(14 < X < 40)

= P(– .5 < Z < 0.83)

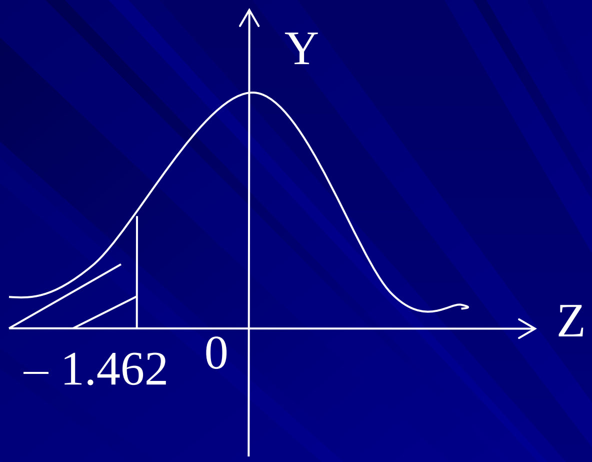
= Φ(0.83) – Φ(– 0.5)

= 0.7969 – 0.3085

= 0.4884

(iv) $P(X < 10)$

$$x = 10 \Rightarrow z = \frac{10 - 30}{12} = -1.67$$



$$P(X < 10) = P(Z < -1.67)$$

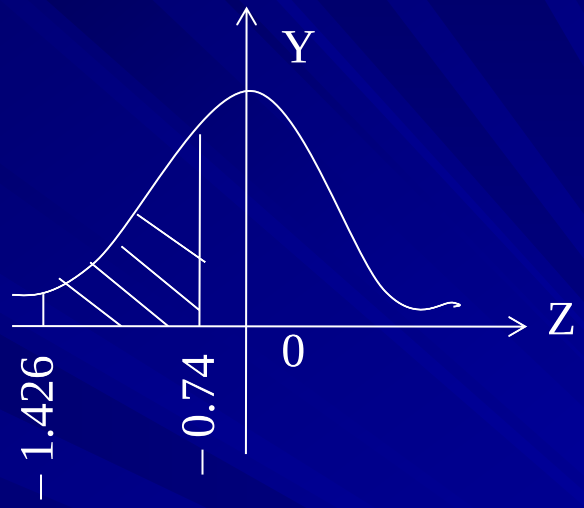
$$= \Phi(-1.67)$$

$$= 0.0475$$

(v) $P(10 < X < 20)$

$$x = 10 \Rightarrow z = \frac{10 - 30}{12} = -1.67$$

$$x = 20 \Rightarrow z = \frac{20 - 30}{12} = -0.83$$



$P(10 < X < 20)$

$= P(-1.67 < Z < -0.83)$

$= \Phi(-0.83) - \Phi(-1.67)$

$= 0.2033 - 0.0475$

$= 0.1558$

Suppose the average length of stay in a chronic disease hospital of a certain type of patient is 60 days with a standard deviation of 15. If it reasionable to assume an approximately normal distribution of lengths of stay, find the probability that a randomly selected patient from this group will have a length of stay; (i) greater than 50 days (ii) Less than 30 days (iii) Between 30 days and 50 days (iv) Greater than 90 days.

X be the length of stay in a chronic disease hospital

$$X \sim n(60, 15^2)\ ,\ Z \sim n(0,1)$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 60}{15}$$

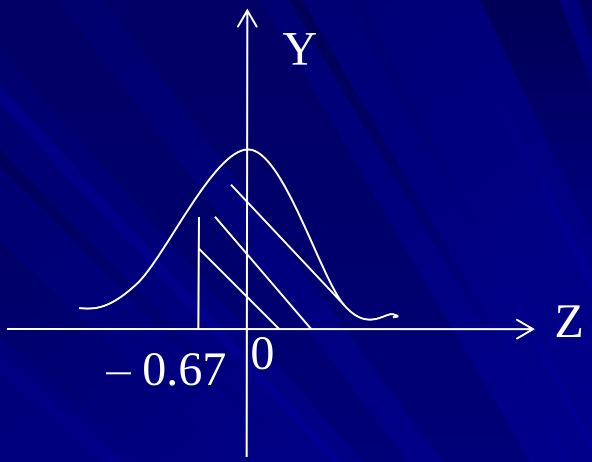
(i) P (X < 50)

$$x = 50 \Rightarrow z = \frac{50 - 60}{15} = -0.67$$

$$P(X > 50) = P(Z > -0.67)$$

(i) $P(X > 50)$

$$x = 50 \Rightarrow z = \frac{50 - 60}{15} = -0.67$$



$$P(X > 50) = P(Z > -0.67)$$

$$= 1 - P(Z < -0.67)$$

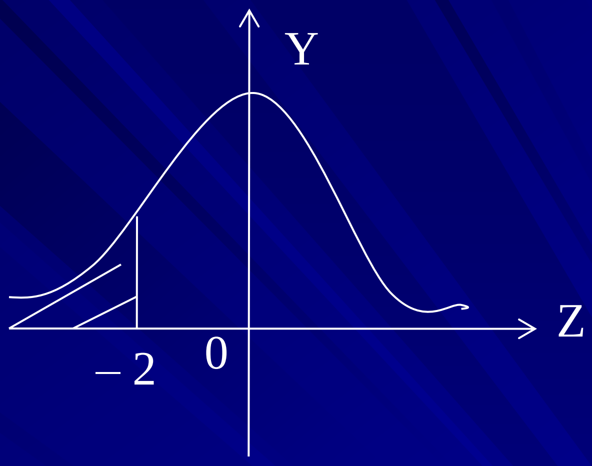
$$= 1 - \Phi(-0.67)$$

$$= 1 - 0.2514$$

$$= 0.7486$$

(ii) P (X < 30)

$$x = 30 \Rightarrow z = \frac{30 - 60}{15} = -2$$



P (X < 30) = P (Z < – 2)

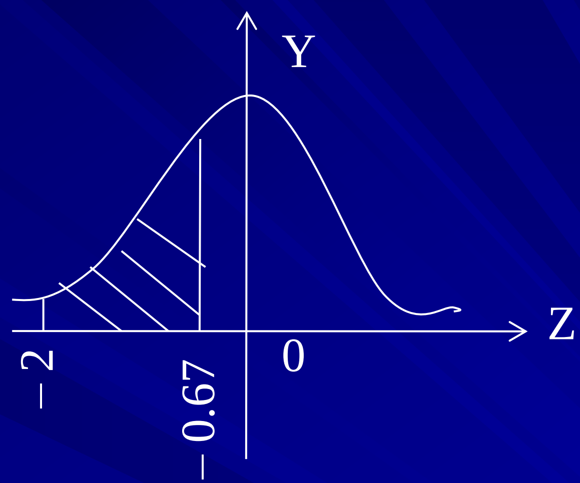
= Φ (– 2)

= 0.0228

(iii) $P(30 < X < 50)$

$$x = 30 \Rightarrow z = \frac{30 - 60}{15} = -2$$

$$x = 50 \Rightarrow z = \frac{50 - 60}{15} = -0.67$$



$P(30 < X < 50)$

$= P(-2 < Z < -0.67)$

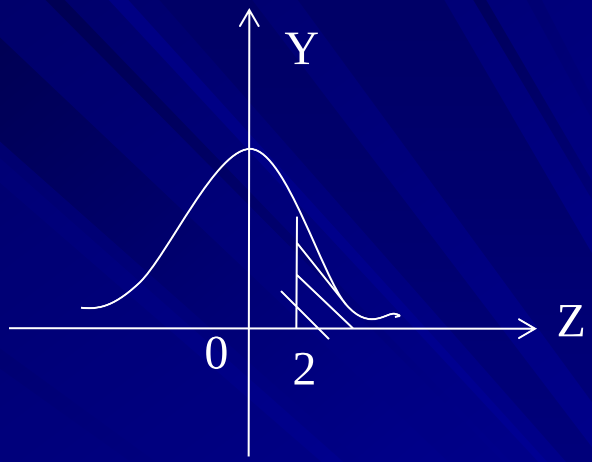
$= \Phi(-0.67) - \Phi(-2)$

$= 0.2514 - 0.0228$

$= 0.2286$

(iv) $P(X > 90)$

$$x = 90 \Rightarrow z = \frac{90 - 60}{15} = 2$$



$$P(X > 90) = P(Z > 2) = 1 - P(Z < 2)$$

$$= 1 - \Phi(2)$$

$$= 1 - 0.9772$$

$$= 0.0228$$